

1. Prove that the following functions are NOT onto:

(a)  $f : \mathcal{P}(\{1, 2, 3, \dots, 100\}) \longrightarrow \mathcal{P}(\{1, 2, 3, \dots, 100\})$  via  $f(X) = X \cap \{1, 2\}$

(b)  $f : (0, 1] \longrightarrow [1, \infty)$  via  $f(x) = \frac{3}{x}$

2. Prove that the following functions ARE onto:

(a)  $f : \mathcal{P}(\{1, 2, 3, \dots, 100\}) \setminus \{\emptyset\} \longrightarrow \{1, 2, 3, \dots, 100\}$  via  $f(X) =$  the smallest member of  $X$

(b)  $f : \mathbf{R} \longrightarrow [-16, \infty)$  via  $f(x) = x^2 - 8x$

3. Prove that the following functions are NOT one-to-one:

(a)  $f : \mathcal{P}(\{1, 2, 3, \dots, 100\}) \longrightarrow \mathcal{P}(\{1, 2, 3, \dots, 100\})$  via  $f(X) = X \cup \{1, 2\}$

(b)  $f : \mathbf{R} \longrightarrow [-16, \infty)$  via  $f(x) = x^2 - 8x$

4. Prove that the following functions ARE one-to-one:

(a)  $f : \mathbf{R} \longrightarrow \mathbf{R}$  via  $f(x) = \frac{3x - 8}{x}$

(b)  $f : \mathbf{R} \longrightarrow \mathbf{R}^+$  via  $f(x) = 2^x \cdot 3^{2x+1}$