

Prove the following via mathematical induction:

1.  $1^2 - 2^2 + 3^2 - \dots + (-1)^{n+1}n^2 = \frac{(-1)^{n+1}n(n+1)}{2}$  for all  $n \in \mathbf{Z}^+$

2.  $\prod_{i=1}^n (2i-1) = \frac{(2n)!}{n!2^n}$  for all  $n \in \mathbf{Z}^+$

3.  $7 \mid (3^{2n} - 2^n)$  for all  $n \in \mathbf{Z}^+ \cup \{0\}$

4.  $9 \mid (4^{3n} - 1)$  for all integers  $n \geq \underline{\quad? \quad}$ , where you determine the correct “base case”

5.  $6n + 8 \leq 7n$  for all integers  $n \geq \underline{\quad? \quad}$ , where you determine the correct “base case”

6.  $5 \cdot 2^n + 1 \leq 3^n$  for all integers  $n \geq \underline{\quad? \quad}$ , where you determine the correct “base case”