Math 235 - Dr. Miller - Discussion/Participation #4 - Fall 2024

Below is one of the attempted proofs given in Problem #12 on p.51 of our text. Critique what is done well and what is missing or incorrect. Also try to identify what style of proof was originally attempted, and whether you would have chosen a different style

12. (d) Suppose a, b, and c are integers. If a divides both b and c, then a divides $b + c$.

    "Proof." Assume that a does not divide $b + c$. Then there is no integer k such that $ak = b + c$. However, a does divide b, so $am = b$ for some integer m; and a divides c for some integer n. Thus $am + an = a(m + n) = b + c$. Therefore $k = m + n$ is an integer satisfying $ak = b + c$. Thus, the assumption that a does not divide $b + c$ is false, and a does divide $b + c$.

13. Now examine each statement below, and list all proof styles we've covered that would be good choices:

    (a) Prop. - Let $m \in \mathbf{Z}$. If m^2 is odd, then m is odd.

    (b) Prop. - Let $t \in \mathbf{R}$. If $t \notin \mathbf{Q}$, then $5t \notin \mathbf{Q}$.

    (c) Prop. - Suppose $x, y \in \mathbf{Z}$. If x and y are even, then $x + y$ is even.

    (d) Prop. - Let $m, n \in \mathbf{Z}$. If $m^2 + n^2$ is even, then m and n have the same parity.

    (e) Prop. - Let $a, b \in \mathbf{Z}^+$. Then $(a + 1)|b$ and $b|(b + 3)$ if and only if $a = 2$ and $b = 3$.