

A biconditional statement uses the phrase “if and only if” somewhere in the middle of the sentence. You know already that the “AND” aspect creates the conjunction of TWO different conditional statements.

The easiest way to recognize those two statements is to just use the word “if” in place of the entire “if and only if” originally, versus using just the phrase “only if” in that same place. That’s really all there is to it.

I call these two conditional statements the “if” direction versus the “only if” direction, depending on which word/phrase is used.

Practice writing these two directions in the statements below - do NOT rearrange any other words in the statements! Also attach the \Leftarrow or \Rightarrow notation that would go with each direction in a two-part proof.

1. $x + y$ is even if and only if x and y have the same parity.

“If” direction (and arrow):

“Only if” direction (and arrow):

2. $p|ab$ if and only if $p|a$ or $p|b$.

“If” direction (and arrow):

“Only if” direction (and arrow):

Some people confuse themselves by insisting on using letter p for the first half of of a biconditional statement (that is, the part that comes before the “if and only if”) and q for the second half. But that will get tangled fast if you also insist that variable p must always represent a hypothesis.

BEWARE! Just as we can use variables other than x in algebraic equations, so we can use variables other than p for a hypothesis, or for the “front” of a biconditional statement. It’s the “if” location and the “only if” location in a statement that identify your hypothesis or conclusion. The logic cares nothing about what you read first versus second.

Only our use of artificial NOTATION (like $p \Leftarrow q$) is fussy about left versus right, and that’s why WE must take charge of the notation (choose the correct arrow label for a proof direction), rather than letting notation try to force its limitations on how we read and understand a verbal sentence.