Math 235 - Dr. Miller - Summary #1: Corrections to Proofs about Relations in Book

Some of the proofs in Section 3.1 of our text are not up to the usual standards you've seen in previous reading. This can sometimes happen when a book has multiple authors, as ours does. Here are **CORRECTED** versions of two proofs that:

(1) Meet our established practices in OUR course this semester, and

(2) Set you up for better success meeting expectations in later courses.

Proofs of (i) and (ii) in the Example on pp.155-156

All is fine with these two proofs EXCEPT that they should not be "hiding" their choice of a candidate within parentheses: that choice should be openly stated as an assumption in the proof.

Here are revised ($\supseteq$) directions of these two proofs, where I've also included any NTS, thus, and therefore lines you would expect (these changes are underlined) AND I've corrected that actual FLAW in the logic structure in bold double-underline:

(i)...($\supseteq$) Now suppose that $x \in [-4, 4]$.

(NTS: $\underline{x \in Dom(T)}$, meaning we must find a real number y such that xTy.) ("We must find" $= \exists$, so ok.)

Then $x^2 \leq 16$.

Consider $y = 0 \in \mathbf{R}$. (Notice how I just slipped in one of our "conditions" in advance.)

(NTS: meets all conditions: in $\mathbf{R}$, and also xTy, meaning $x^2 + 4y^2 \leq 16$.)

Then $x^2 + 0 \leq 16$, so $xT0$.

Therefore $x \in Dom(T)$.

(ii)...($\supseteq$) Now suppose that $y \in [-2, 2]$.

(NTS: $\underline{y \in Rng(T)}$, meaning we must find a real number x such that xTy.) ("We must find" $= \exists$, so ok.)

Then $y^2 \leq 4$.

Consider $x = 0 \in \mathbf{R}$. (Again, I slipped in one of our "conditions" in advance.)

(NTS: meets all conditions: in $\mathbf{R}$, and also xTy, meaning $x^2 + 4y^2 \leq 16$.)

Then $0^2 + 4y^2 \leq 16$, so $0Ty$.

Therefore $y \in Rng(T)$.

Example at the bottom of p.159 to top of p.160

All is well until the proof reaches its $\neq$ claim in the very last sentence of the proof. It's a very risky idea in mathematics to claim either of these appearance-based statements:

(1) that sets aren't equal because their membership conditions LOOK different - think about our $5\mathbf{Z} = \{x|x = 10a + 15b \text{ for some } a, b \in \mathbf{Z}\}$ problem earlier in the course, and

(2) that different-LOOKING formulas automatically produce different results (we have $\sin^2 x + \cos^2 x$ versus 1 to highlight that we can't base non-equality just on looks!)

So some rigor is definitely lost in that last sentence, but we can easily fix it. Just change the final sentence to something like,

"Clearly $S \circ R \neq R \circ S$ because $\underline{\underline{(1,2) \in S \circ R \textbf{ since } 2 = 1^2 + 1\textbf{, but } (1,2) \notin R \circ S \textbf{ since } 2 \neq (1+1)^2\textbf{."}}}$

Now we've used a nice, rigorous interpretation of the definition of set equality, which is so much better.

Proof of Theorem 3.1.2(b) on p.160

I would take a big deduction off this proof on an assignment. Proofs must use **WORDS** and sentences, not just notation. This isn't a computer coding class. It's not just the $\exists$ symbols that are the problem. Those square brackets and the shifting of parentheses into and out of them is hard to read and likely confusing just to read for freshmen/sophomores!

The ORDER of their steps is great, and even their chain-style is very nice: they just need to write for a human reader, and not a machine. Here's a fix that simply translates their parentheses, brackets, and lost punctuation into words and comma-separated phrases:

Proof - Suppose that A, B, C, and D are sets, and let R, S, T be as described.

The pair $(x, w) \in T \circ (S \circ R)$ for some $x \in A$ and $w \in D$

iff there exists $z \in C$ where $(x, z) \in S \circ R$ and $(z, w) \in T$

iff there exists $z \in C$ such that there exists $y \in B$ where $(x, y) \in R$ and $(y, z) \in S$, and also $(z, w) \in T$

iff there exists $z \in C$ and there exists $y \in B$ where $(x, y) \in R$ and $(y, z) \in S$, and also $(z, w) \in T$

iff there exists $y \in B$ and there exists $z \in C$ where $(x, y) \in R$, and also $(y, z) \in S$ and $(z, w) \in T$

iff there exists $y \in B$ where $(x, y) \in R$, and there exists $z \in C$ where $(y, z) \in S$ and $(z, w) \in T$

iff there exists $y \in B$ such that $(x, y) \in R$ and $(y, w) \in T \circ S$

iff $(x, w) \in (T \circ S) \circ R$.

Therefore, $T \circ (S \circ R) = (T \circ S) \circ R$.