MATH 235: Foundations of Mathematical Proof - Dr. Miller - Syllabus - Fall 2024, CRN (9075)

Keep this syllabus with your course handouts. You are responsible for reading it entirely.
It is subject to change in extenuating circumstances.

Meeting Times/Place: MWF 1:00-1:50, ECB 011

Course Description/Content: (The Department-approved Student Outcomes are attached also.)

- SRU catalog: This course provides a foundation for writing mathematical proofs. Students will work common proof techniques such as direct proof, proof by contrapositive, and proof by contradiction, along with the logical bases supporting them. In developing their proof-writing skills, students will work with foundational mathematical concepts such as basic set theory, induction, relations between sets, and functions.
- This course builds heavily on material from MATH 131 - Discrete Mathematics.
- Our primary focus is the reasoning necessary to create and understand mathematical proof.
- This course requires you to learn to write formal proofs and is very VERBALLY oriented.

Prerequisites and Follow-Ups:

- Prerequisites: MATH 225 – Calculus I and MATH 131 - Discrete Mathematics
- Mathematics majors must have a C or better in both of these prerequisite courses.
- Follow-up course-work: Mathematics majors must earn at least a C in this course.

Text: *A Transition to Advanced Mathematics* by Smith, Eggen, and St. Andre (8th ed., teal cover)

- The eText is linked in D2L if you have Inclusive Access, but used print copies are okay, too.
- We do NOT use any of the online Cengage homework or add-ons.

Classroom environment:

- I can't allow certain foods/drinks (especially nut-flavored) due to SRU-documented disability.
- See me if these restrictions conflict with your own documented accommodations.
- I teach using at-the-board lecture and a few activity hand-outs. You might also present informally.
- Don't distract yourself with cell phone use or off-topic chatter during class.
- **Protect necessary study time *outside* class.** HW in proof classes requires many hours each week.
  - The expectation in a typical college course is to study 2-3 hours outside class for **each** hour in class, because the US Dept. of Education *defines* ONE credit hour as "One hour of classroom or direct faculty instruction AND a minimum of two hours of out of class student work each week."
  - **For advanced math/science courses, you should probably double/triple that.**
  - Don't expect to complete HW the night before it's due. That won't succeed.
  - Budget enough time: Arrange jobs, family, extracurriculars, etc., to protect the time you need.

Personal Environment: I go by my middle name with friends and Dr. Miller in professional settings, so I am sensitive to what other people want to be called. Please let me know your preferences.

Student Hours and Contact Info: Email: lyn.miller@sru.edu - Phone: 724-738-2878 - Office: VSC 200-B

- Drop-in F2F Student Hours are: **MWF 10:30-11:30 am and F 2-4pm**
- I can make appointments outside of listed office hours, and if my door is open, I'm available "by capture."
- **Course assignments are posted on our web page (not D2L) at granite.sru.edu/∼lmiller .**
  - Each day's post includes: name of topic covered, links to in-class handouts, supporting reading list, list or link of upcoming assigned problems, and due date reminders.
  - I only use D2L for things that need to be private: your grades, copyrighted handouts, etc.
- Email is my preferred communication with you, and here are some general guidelines:
  - I check SRU email every weekday morning, but I am in class/meetings roughly 12:30-5 daily. I can check afternoon emails when I end the day around 5-6pm.

- I try to acknowledge emails that same day, but may need another 1-2 business days to take action, depending on time of day, other responsibilities, and whether anyone else must be looped in.
- Evenings/weekends, I try to model professional boundaries and etiquette: neither you nor I should expect replies to after-hours emails, unless we've made prior arrangements.
- I will notify everyone ASAP about any unexpected issues; I ask that you do the same (absence, etc.).

Grading: $A = 90 - 100\%; B = 80 - 89\%; C = 70 - 79\%; D = 60 - 69\%; F = 0 - 59\%$.

- 600 pts total: HW Score = 120 pts, Discussion = 30 pts, Exams 1-3 = 100 pts each, Final = 150 pts
- Students with SRU-documented class or assessment accommodations should notify me ASAP.

Homework (HW): Turned in weekly, worth 20 points each, best 8 kept (of 10-12) and scaled to 120 course pts

- HW is usually due on Fridays late afternoon; proofs take MUCH time, so start early!
- I give a list with a 5-7 day window before it's due. Try to start that same day.
- Please staple, remove ragged edges, write legibly (or TeX), and leave LOTS of room for me to comment.
- I grade a selection among the problems each time, for correctness and rigor (we'll define this term).
- **Make-ups - typically NONE**, and late only with prior approval (and often a deduction).
  - Keeping your best 8 scores (of 10-12) allows for travel, illness, etc, like earning "personal days" at work; everybody is allowed to miss, but choose wisely.
  - For extenuating or long-term situations, I may approve double-counting a future assignment to replace a missed score, but this requires written, external documentation, a University-recognized circumstance (illness, armed forces, etc.), and meaningful discussion with me ASAP.
- Turning in HW early or via email when you'll be absent is usually approved, but ask first.
- I may also give opportunities to learn from your feedback and present problems to me in my office to re-earn missed points or missed assignments. This will vary, depending on the topic of the assignment.
- You may collaborate on HW or get tutoring help, but **do not COPY** from others, the web/AI, books, etc.
- **Inappropriate collaboration may result in a score of 0 for all involved, regardless of intent.**
- Solutions (sometimes partial) will typically be posted outside my office door and on D2L.
- To honor copyright laws and other restrictions, do NOT share solutions beyond our course.

Discussion/participation (DP): (Subject to change as we go) Occurs as announced, worth 3 points each.

- Your best TEN scores are kept for the course, for 30 course points total.
- DP may be group proof analysis or solo at-the-board explanations. I hope to involve everyone, often.
- Scored just on participation/effort, you should be able to earn perfect scores every time.
- DP are informal, low-intensity chances to explore potential errors, before it matters on HW or exams.
- **Make-ups - NONE**. I plan to give everyone at least 10 opportunities, or else adjust case-by-case.

Mid-Term Exams: Worth 300 points total: 100 each for Exam #1, #2, #3, no collaboration

- Midterm exams cover everything since the last exam, but I also give a printed Topics List 1 week in advance.
- By nature, this course is rather cumulative, so be prepared to use earlier skills too.
- Exams usually have a small, individualized take-home portion due the next class day also.
- Answers without work/support do not earn full credit. Justifications and proofs must be rigorous.
- Justifications/work/proof are graded on correct MATH knowledge, notation, reasoning, and style.
- Honesty and integrity are expected; violations may result in a 0 for all involved.
- Tentative dates are on the course calendar on our web page.
- Students with SRU-approved accommodations must submit ODS requests one week in advance.
- Exam solutions are posted outside my office door and on D2L after grading.
- **Make-up Exam Scores** require a documented reason and meaningful efforts to contact me in advance.
  - If I approve your make-up score, you won't actually take an immediate replacement exam, adding stress to a likely already stressful return to class.
  - Instead, your percentage on the relevant material when it shows up on the cumulative Final Exam will fill in for your missed midterm exam score. Study that doubled material extra well!

Final Exam: cumulative, worth 150 course points total, no collaboration, honesty/integrity expected

- You MUST take the Final at the scheduled time: **THURSDAY, Dec. 12, 1:00-3:00** (in this room)
- Students with SRU-approved accommodations must submit exam paperwork to me one week in advance.
- Final exams are not returned to you, and grades typically take 3-5 days to finalize.

Attendance and Help:

- I take attendance daily, but it does NOT count toward your grade.
- If you are absent, get the notes from a **classmate**. This is **YOUR** responsibility. (I lecture from an outline, so I don't *have* actual, study-able notes or PowerPoints to xerox if you're absent.)
- Handouts and announcements are available via my web page **granite.sru.edu/∼lmiller**.
- **The Department does not in general provide peer tutoring for this course.**
- You can get help from me during office hours (above) or by appointment, and by working together.
- SEEK HELP EARLY AND OFTEN!

Student Outcomes - Math 235 - Foundations of Mathematical Proof

SRU Department of Mathematics - effective Fall 2022

1. Students will identify and analyze the logical structure of mathematical statements.
2. Students will comprehend and analyze formal mathematical proofs.
3. Students will write coherent mathematical proofs using the techniques of direct proof, contrapositive, contradiction, and induction.
4. Students will write proofs involving naive set theory, the formal definition of functions, and relations.

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Important University-wide policy statements:

Title IX: Slippery Rock University and its faculty are committed to assuring a safe and productive educational environment for all students. In order to meet this commitment and to comply with Title IX of the Education Amendments of 1972 and guidance from the Office for Civil Rights, Title IX requires University faculty members to report incidents of sexual discrimination, including sexual violence, shared by students to the Universitys Title IX Coordinator. Accordingly, if a student shares information about any incidents of sexual discrimination or sexual violence during a classroom discussion, in a writing assignment for a class, or in other contexts, faculty must report that information to the Title IX Coordinator. This information will only be shared with the Title IX Coordinator, who is the individual on campus designated to respond to reports of discrimination or sexual violence. While the Title IX Coordinator is not a confidential source of support, they will address matters reported to them with sensitivity and will keep your information as private as possible.

Additionally, faculty members are obligated to report sexual violence or any other abuse of a student who was, or is, a child (a person under 18 years of age) when the abuse allegedly occurred, to the person designated in the Universitys Protection of Minors Policy.

Information regarding the reporting of sexual violence and the resources that are available to victims of sexual violence is set forth at: https://www.sru.edu/offices/human-resources-and-compliance/sexual-misconduct-and-title-ix-resources

Slippery Rock Universitys Title IX Coordinator is Karla Fonner and she can be reached at karla.fonner@sru.edu; 724-738-2953, or you can submit a Care Referral with the details of the situation.

Non-discrimination: Slippery Rock University of Pennsylvania does not discriminate on the basis of race, color, sex, sexual orientation, gender identity, gender expression, national origin, religion, age, disability, or veteran status in its programs or activities in accordance with Title IX of the Educational Amendments of 1972, the Americans with Disabilities Act of 1990, Section 504 of the Rehabilitation Act of 1973, Title VII of the Civil Rights Act of 1964, and other applicable statutes and University policies. www.sru.edu/offices/diversity-and-equal-opportunity/notice-of-non-discrimination .

More Info about this kind of course and its philosophy

- **What This Course Is About, Why You Need It, and How It's Taught**

  Content and Rationale: To mathematicians, the word "proof" has a very deep meaning that is often difficult to define. A proof is not just a list of computations that ends up with some numeric result or in some desired form of symbols. Rather, **a proof is a clear, connected set of mostly verbal statements (though often mingled with equations or other notation) that justifies why some particular behavior will happen or some property will be true**. Writing a proof is the art of nailing down a guarantee in some sense about things that are not always specific. The purposes of this course are to help you (1) identify what the "guarantee" really is in any given reasoning setting, (2) recognize sentences or notation that MUST be included in order to meet that aim of "connecting" your statements, and (3) sharpen your attention to verbal and notational detail that will make your proofs "clear." FORMAL use of language and notation aren't just nit-picky choices by one instructor vs. another, but rather are the true building blocks of mathematical proof in all courses.

  It can be tough to make the switch from computational-type thinking and work that served you well in courses like algebra and calculus, to grappling simultaneously with both the information that you need to say AND the way you need to say it, and many math students say that writing proofs is one of the hardest things they try to learn. It is also extremely satisfying, though, like learning any new but challenging skill: reading and writing proofs opens the doors to understanding WHY so much of what we were taught in grades K-12 really is true or has to work in a particular way. Proof makes SENSE of rules that many people just grudgingly accept as somebody's annoying whim. And mathematics at its most basic could be described as making sense (predictions, rules, guarantees, truth!) of the world around us.

  Pedagogy: In any proof-based course, YOU will definitely be writing lots of original proofs. On my end, I still lean toward the lecture-as-model style of teaching, with a lot of discussion thrown in. I teach in this manner because I hope it lets you slowly work through each step of a proof as we build general styles that will be needed in many of your further courses. This course is not so much about learning specific math ideas, though, as it is about learning specific reasoning *patterns*. So to see enough examples to recognize a pattern, lecture can give you a model for how the material is logically organized and how it flows most clearly and even elegantly. However, there's also a potential weakness to lecture: during a polished lecture, even an excellent one, you typically don't get to see the professor DOING mathematics; you only get to see him/her PRESENTING mathematics. Most students may not realize that there's a difference. So we math professors face a dilemma: how do we in the same course demonstrate often what a good finished product of a proof should look like while also demonstrating how to arrive at one from scratch, with all the fits and starts, back-tracking and just plain re-booting that the process entails? There are a lot of valuable approaches and philosophies for attempting this, but in the end, most math faculty agree that we should each teach according to our own strengths. For me, that is still presenting proofs and techniques at the board with lots of discussion about the rationale for each step, and expecting you to internalize that rationale (with note-taking!) so that you can recognize and use it in similar situations in this class and others. I hope this teaching technique will be successful in helping you to learn the material and the broader reasoning and proof skills the course is used to develop.

  Cognitive Environment: It's very important to stay focused during class, so avoid distracting side behaviors. I encourage collaboration in and out of class, but collaborating isn't the same as copying (which is actually cheating/plagiarizing, believe it or not) from a partner nor even having an entire group agree on a common response. You should always arrange your own work in your own way and give explanations/proofs in your own words, for when you don't, you set yourself up to do poorly in settings where you are expected to work alone, like on exams or in your own job. Also, remember that **while one professor may allow collaboration in a course, another may not**, or someone may allow collaboration on some tasks but not others in the same course. Always check to see whether working together is allowed in each of your classes. (See below about AI use.)

  Sources of Help: Remember, you are allowed and encouraged to study together for this course, but make sure you write up assignments in your own words. Please refrain from using solution manuals or web searches to look up answers or solutions. Doing so deprives you of the opportunity to improve your own skills. (By the way, generative AI sites write TERRIBLE proofs! They probably wouldn't pass my course, so don't trust them.) And as I've often been told that my exams require students to "think on their feet," losing out on genuine individual practice will probably put you at a keen disadvantage for exams.

  Since this is essentially an upper-level course, the Department does NOT provide peer tutoring for it. A peer tutor doesn't yet have the sufficiently broad and deep views of proof-based mathematics needed to be effective.

It would put unfair pressure on them to expect that. If you need help, please come see me during office hours, or we can make an appointment. For this course in particular, I have an "open door" policy, meaning that I'm willing to help you outside official office hours if I am free. So if you need help but it's not my scheduled office hours, you can certainly ask if I can help. If I'm not in the middle of another commitment, I will take time to work with you. If you see my door physically open, it usually means that I know I can interrupt other tasks to focus on you.

Criteria/Expectations: HW and exam scores reflect how well you understand the course content, and that means concepts – ideas, relationships, vocabulary, proof structure, etc. – not just algorithms that move numbers and symbols around. Therefore, on homework and exams, I expect that you will, among other things:

Show correct, complete work on computationally-based problems: I often award partial credit for some correct work even with a wrong answer; conversely, if your work for a problem is wrong or incomplete, you will earn very few points even if you got a correct value or expression in the end. Also be sure you actually answered the question or drew a conclusion. For instance, if a problem asks you to find a total number of people, and you only tell me - separately - about how many adults and how many children, you haven't completed the task.

Give clear verbal justifications when asked, and write rigorous proofs: Verbal justifications and proofs are the main content of this course. Students are sometimes surprised by the very high quality I require in order to earn full credit on them. Your informal justifications or explanations *will* be verbal/words, not just lists of computational steps. You must aim for the calibre of a future professional supporting the method or explaining the material, NOT of a high-schooler showing their scratchwork or describing their thinking. You should also be showing or writing about the necessary MATH in a problem, not about your internal train of thought. Unhelpful statements such as "I just kept trying numbers until it worked" or disorganized steps whose order makes sense only to you are unlikely to be worth many points as a justification or explanation.

The most frequent verbal component of this course is proof-writing. Proofs require **rigor, a term that means (1) you have no errors in the logic connecting your ideas/statements and (2) you have not skipped any steps in reporting those connections**. Recognizing what rigor is, and what it does or does not look like, are skills this course helps you learn, so I will provide plenty of coaching. Be aware that the ultimate goal is to have you develop a sense of when you have been rigorous in writing a proof, so that you avoid logic errors and skipped reasoning in later courses.

Apply critical thinking: Fully comprehend instructions: what I ask for IS what you'll get points on, no more, no less. I see two kinds of mistakes here: first, sometimes students don't think about or don't understand the ordinary English vocabulary. For instance, the definition of "explain" implies to use words, so if you merely list equations or computations with no verbal discussion to connect them, you will earn 0 points because you didn't give an actual explanation. Second, I assess for you're-going-into-this-profession-someday skills. So for example if a question says to demonstrate one technique but you show another, again you will get few points because the point of the question is not just to get a final answer any old way, but to demonstrate the ability to adapt your approach and have a large and varied toolbox of skills (necessary from a professional).

Behave with integrity: In your professional life, the people you deal with will expect you to be honest with them, maintain standards, and not cut corners. Establishing those traits early and solidly is important, so I will expect such behavior of you in this class. Being honest and not cutting corners means doing your own work on assignments, not just copying from a friend or worse, cutting-and-pasting from the web/AI - that's plagiarism! It means not trying to give yourself an unfair or impermissible advantage on exams or quizzes through things like crib sheets, web helps, etc. Being honest obviously includes not giving a false or unfairly exaggerated reason to try to get a make-up or extension on a task. Maintaining standards means not doing someone else's work for them while letting them turn it in as their own. SRU's policy on academic integrity is at rockpride.sru.edu/policies/#search=integrity . The University expects that students will demonstrate their mastery of subject matter (in our case, skills, outcomes, and knowledge in our course) in an honorable and straightforward manner.