Math 235 - Dr. Miller - Exam #1, In-Class Portion, Spring 2023 - Friday, Feb. 17, 2023

Follow directions carefully; work in the space provided. This in-class part of the exam is worth 85 points.

There are 3 full proofs. Each is marked with **[P]**. There are also 3 launches, each marked **[La]**.

1. *[15 pts - 5 each]* Precisely negate each statement below. Don't worry about whether the statements are true or not.

   (a) $x^2 - 5x + 6 \geq 0$ only if $x \leq -2$ or $x \geq 3$.

   (b) $y = 7$, and $x^2 = 9$ if $|x| = 3$.

   (c) There exists $n \in \mathbf{Z}$ where $nx > 1$ for all $x \in \mathbf{R}^+$.

2. (a) *[2 pts]* Write the logical equivalence governing proof by cases.

   (b) *[2 pts]* Write the logical equivalence governing two-part proof (of biconditional statements).

3. *[12 pts]* **[P]** Use the formal definition of $<$ to write a rigorous direct proof of the statement below. (I'll give you the definition, for a deduction.)

Proposition: Let $p, q, x, y \in \mathbf{R}$*. If* $p < q$ *and* $x < y$*, then* $p + x < q + y$*.*

4. *[12 pts]* **[P]** Prove rigorously, using direct proof. (I'll give a hint, for a deduction.)

Let $x, y \in \mathbf{Z}$ *have the same remainder on division by* 3. *If that remainder is not* 0, *then* $3|(xy - 1)$.

5. *[12 pts]* **[P]** Prove by any meaningful style. (Surprise: $\geq$ algebra allowed, but for a deduction.)

Proposition: Let $t \in \mathbf{R}$. *If* $|t| \geq 5$, *then* $2t + 8 \neq 0$.

(Remember that formal $<$ definition is NEVER required for concrete numbers.)

6. *[16 pts - 8 each]* Consider this Proposition: *Let* $x, y \in \mathbf{Z}$. *If* xy *is even, then* x *is even or* y *is even.*

(a) **[La]** Write the launch, up to and including one meaningful sentence BEYOND the NTS line, of a proof by contrapositive. **Do NOT complete the proof.**

(b) **[La]** Write the launch, up to and including one meaningful sentence BEYOND the NTS line, of a proof by "or conclusion" style. **Do NOT complete the proof.**

7. Consider this Proposition: *Let* $m \in \mathbf{R}$*, and let* $f(x) = \arctan x$ *and* $g(x) = mx - \frac{\pi}{2}$*. The graphs of* f *and* g *DON'T intersect if and only if* $m = 0$*.*

   (a) *[2 pts]* Ignoring the universal hypothesis for now, write the "if" direction in unchanged order.

   (b) **[La]** *[8 pts]* Including the universal hypothesis, write the launch, up to and including one meaningful sentence BEYOND the NTS line, of a proof by contradiction for the "if" direction. **Do NOT complete the proof.**

8. *[4 pts]* Formally state the Fundamental Theorem of Arithmetic (FTA).