Math 235 - Dr. Miller - Topics List for Exam #1, Spring 2017 - **Friday, Feb. 24, 2017**

Prepare by studying the topics listed below, in conjunction with your notes, text, any handouts, and graded and ungraded HW problems. You will be more successful if you strive to master the concepts in general, rather than simply memorizing specific examples that we have already done. Studying together is a plus.

Non-Proof Tasks:

1. Identify hypothesis or conclusion in a conditional statement that's written using: if-then, "trailing" if, only if (including "split" only if), necessary, sufficient, implies.
   (a) Your response should be a stand-alone sentence, and it CAN'T keep any of the conditional words: if, then, only, necessary, or sufficient.
2. Give SIMPLIFIED verbal negations of given statements that contain: conditional, if and only if (iff), and, or, for all/each/every/any, there exists/is, at least, greater/less/equal.
   (a) You must "swap" quantifiers ("not all" or "there doesn't exist" is incorrect in this case).
   (b) You must also use De Morgan's Laws where appropriate.
   (c) Beware situations where the quantifier phrase is trailing, as in "$x = 2n + 1$ for some $n \in \mathbf{Z}$."
   (d) Remember that the negation of a conditional statement is NEVER still conditional.
3. Be familiar with the notations $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, and similar variations for $\mathbf{Q}$ and $\mathbf{R}$.
4. Understand a set defined using set-builder notation. List a few elements when asked.
5. State FORMAL definitions of: rational, even, odd, Division Algorithm, divides.
   (a) Formal definitions not only use variables, but often VERBAL quantifiers (not $\forall$ or $\exists$) and buffer words. Don't use commas in place of buffer words.
   (b) Formal definitions always have universal hypotheses too. You must state those.
   (c) Giving a formal definition in math is not just expressing an idea of the meaning of a word, but rather a PRECISE phrasing that perfectly control all the associated logic. MEMORIZE clear, complete definitions in order to earn full credit.

Proof Tasks: Using direct, cases (including "wlog"), "or conclusion," contrapositive, contradiction.

1. Some problems require you to use MY choice of proof type; others may be left to YOUR choice.
2. Practice so that when the choice is yours, you can decide rather quickly.
3. Most problems will require full proof, but there may be shorter tasks where I ask you only to write what we assume and what we "NTS" for MY choice of proof, and then stop.
4. Statements may be about even, odd, rational, remainders, divisibility/divides, set-builder sets.
   (a) I'll require even, odd, rational, divides proofs, to use definitions, not childhood knowledge.
5. You should also be comfortable using basic algebra concepts such as equation/inequality solving, equations of lines or circles, etc.
6. Remember these proof components:
   (a) You MUST write your explicit assumptions at the outset, including universal hypotheses.
   (b) You MUST be clear that you have proved what was asked, via the "exit move" sentence.
   (c) You must write in SENTENCES. Proofs are never just a string of equalities or algebraic expressions alone. Such things always appear in the context of words describing what they represent/what you are doing with them.

General Advice:

1. There will be 3-5 complete proofs on the exam, with several smaller tasks.
2. Given the time constraint, you'll find that some correct proofs require only a few sentences.
3. Some statements may be similar to examples you've already seen, but others will not be (else I'm only testing your ability to imitate, rather than actually create your own proof).
4. By their very nature, proof exams require creativity.
   (a) When required to be creative under a time constraint, it's natural to feel lost, rushed, or even blind-sided. Try not to panic, but keep working to show me as much of your skills as possible.
   (b) I like to give partial credit, so include as many *meaningful* ideas as possible, even if you're stuck.
   (c) Putting the "exit move" and other final sentences on the end typically earns some credit, regardless of whether you have a whole in the middle of your proof.
   (d) Again, practice *A LOT* (there are plenty of problems left in the text!), so that you feel comfortable choosing a proof type AND making some progress on any statement you're asked to prove.