

Prepare by studying the topics listed below, in conjunction with your notes, text, handouts, and graded/ungraded HW problems. You will be more successful if you strive to master the concepts in general, rather than simply memorizing specific examples that we have already done. Studying together is a plus.

Skills from Exam #1 are still needed throughout the course. These include:

1. Identify hypothesis or conclusion in a conditional statement that's written using: if-then, trailing if, only if (including "split"), necessary, sufficient, implies, when/whenever.
2. Write contrapositives. Convert between universal, conditional, and universal conditional.
3. Negate: conditional, and, or, for all/each/every/any, there exists/is, at least one, greater/less/equal, etc.
4. Be familiar with the notations \mathbf{Z} , \mathbf{Z}^+ , \mathbf{Z}^- , similar variations for \mathbf{Q} and \mathbf{R} , and set-builder notation.
5. Use FORMAL definitions: rational, even, odd, Division Algorithm, divides, Linear Combination Thm.
6. Know the different synonyms within divisibility: divides, factor, multiple, divisor, divisible by.

Non-Proof Tasks: There are very few this time.

1. Formally state: Division Algorithm, Linear Combination Theorem, Fundamental Theorem of Arithmetic.
2. Formally define: even/odd function, increasing function.

Partial Proof Tasks:

1. Verbally describe proof techniques by telling what to assume and what you need to show when proving:
 - (a) "If P , then Q " directly or by contrapositive (in general or for a specific statement)
 - (b) Any statement by contradiction (so remember how to negate EVERYTHING)
 - (c) "If P or R , then Q ," "If P , then T or S " (in general or for a specific statement)
 - (d) " $\forall x, P(x)$ ", constructive " $\exists x, P(x)$ " (in general or for a specific statement)
 - (e) The SEPARATE directions of a biconditional statement directly, by ctp, or by contradiction
 - i. Beware: I refer to these as "if direction" and "only if direction," rather than using notation.
2. As on HW #5, decide which style among direct or contrapositive is best for a given statement, and justify.

Full Proof Tasks: Write COMPLETE proofs of various statements.

1. Prepare to use all TEN of the styles we've covered:
 - (a) Direct proof, proof by contrapositive, proof by contradiction
 - (b) "For all" proof, constructive "there exists" proof, disproof using counterexamples (cex)
 - (c) Two-direction proof, "or conclusion" proof, uniqueness proof, proof by cases
2. Some problems require you to use MY choice of proof type; others may be left to YOUR choice.
 - (a) Practice so that when the choice is yours, you can decide rather quickly.
3. Prove that a given root is irrational by lowest terms argument or FTA.
4. Remember that your NTS line often helps you decide how to continue in a deep proof.
5. As on Exam #1, statements may be about:
 - (a) Parity, rationality, remainders, divisibility, set-builder sets, concepts we've used in this class
 - (b) Basic algebra concepts such as equation/inequality solving, equations of lines or circles, etc.
6. Also prepare for function behavior: graph intersection, even/odd/increasing functions, etc.
7. Beware algebra mistakes, especially:
 - (a) Bad factoring/equation solving; multiplying inequalities without confirming positive/negative
 - (b) FTA without checking positive/negative
 - (c) Bad directions when chaining several inequalities ($a \leq x > c$ is BAD!)
8. Remember these proof components:
 - (a) You MUST write your explicit assumptions at the outset, including universal hypotheses.
 - (b) You MUST be clear that you have proved what was asked, via the "exit move" sentence.
 - (c) You must write in SENTENCES. (For instance, algebra alone \neq proof.)

General Advice: We will plan on adding extra time at the end of the period.

1. There will be 3-5 complete proofs on the exam, with several smaller tasks.
2. When required to be creative under a time constraint, it's natural to feel lost, rushed, or even blind-sided. Try not to panic, but keep working to show me as much of your skills as possible.
3. I like to give partial credit, so include as many *meaningful* ideas as possible, even if you're stuck.
4. Putting the "exit move" and other final sentences on the end typically earns some credit, regardless of whether you have a hole in the middle of your proof.
5. Again, practice *A LOT* (there are plenty of problems left in the text!), so that you feel comfortable choosing a proof type AND making some progress on any statement you're asked to prove.