Math 235 - Dr. Miller - Topics List for Exam #2, Spring 2024 - **Friday, Mar. 22, 2024**

Prepare by studying the topics listed below, in conjunction with your notes, text/readings, any handouts, Short Practice, and graded and ungraded HW problems. You will be more successful if you strive to master the concepts in general, rather than simply memorizing specific examples that we have already done. Studying together is a plus.

Skills from Exam #1 and Discrete Math are still needed throughout the course. These include:

1. Identify hypothesis, conclusion in a conditional statement that's written using: if-then, trailing if, only if.
2. Write contrapositives. Recognize "if" versus "only if" directions in biconditional statements.
3. Negate: conditional, and, or, for all/each/every/any, there exists/is, at least one, greater/less/equal, etc.
4. Translate between conditional and universal statements as you find useful.
5. Be familiar with the notations $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, similar variations for $\mathbf{Q}$ and $\mathbf{R}$.
6. Use FORMAL definitions/statements: rational, even, odd, Division Algorithm, divides, FTA, intervals.
7. Know synonyms for divides (factor, multiple, divisor, divisible by) and how remainders work.

Non-Proof Tasks: Formal statements/definitions have hypotheses. Don't forget to write them!

1. Formally state: defn of $A \subseteq B$, $A \nsubseteq B$, $A \cup B$, $A \cap B$, $A \setminus B$, $A \times B$, A^c, $n\mathbf{Z}$, $\mathcal{P}(S)$.
2. Informally define "unique" (in math); describe (in words) ways to prove it.
3. Answer questions about set notation, as on HW #6: elements, power sets, subsets, operations.

Partial Proof Tasks: "Launches" There will be several problems like this.

1. Show just the START of a proof by giving: assumptions, NTS, and ONE additional sentence.
   (a) I will specify whether you're to go one sentence beyond a first, second, or even third NTS.
2. I'll ask you to use MY choice among proof styles for different types of statements:
   (a) Conditional statements: Direct (including cases), by contrapositive, or by "or conclusion" style
   (b) Any statement: by contradiction (so remember how to negate EVERYTHING)
   (c) Direct "for all" proofs, constructive "there exists" proofs
      i. For the constructive proof, you don't need to know the exact candidate; you can just write "Consider t" LITERALLY as your assumption and then move to NTS.
   (d) Subsetness, set equality, or not-a-subset proof
   (e) Set identities: "chain-style" proof OR ordinary set equality proof
   (f) The SEPARATE directions of a biconditional statement directly, by ctp, or by contradiction
      i. Remember: I refer to these as "if direction" and "only if direction," rather than using notation.
3. Statements may use unfamiliar concepts, but don't panic - you will NOT write full proofs about those!

Full Proof Tasks: Write COMPLETE, rigorous proofs of various statements.

1. Emphasis will be on these **THREE** styles/types of statements:
   (a) Quantifier proofs: direct or $\nexists$ "for all," constructive "there exists," both quantifiers mixed,
      i. In a constructive "there exists" proof, your candidate might be a concrete value, or it might be a formula. Scratchwork inside parentheses is fine in order to find that candidate.
   (b) Set proofs: subset/not/=, set identities (two-part or chain-style), general set proofs.
      i. Prepare for general set proofs to use the operations: $\cup$, $\cap$, $\setminus$, $\times$, complement.
   (c) Full uniqueness proof appears only on the take-home, but see above about describing the structure.
      i. **There is ONE take-home problem: an existence/uniqueness proof combined.**
2. You must still navigate all earlier proof styles: direct, cases (including WLOG), "or conclusion," ctp, $\nexists$.
3. We've also seen that styles may end up mixed now.
4. I may require you to use MY choice of proof type, or leave that choice to YOU.
   (a) Practice so that when the choice of proof style is yours (not mine), you can decide rather quickly.
5. Your NTS line, use of defns, or looking at past assumptions helps you continue when stuck.

6. Propositions can be about sets (of course) OR non-set ideas, such as these earlier or familiar concepts:
   (a) Parity, rationality, remainders, divisibility, other concepts we've used in this class
   (b) Algebra/pre-calc concepts such as equation/inequality solving, equations of lines/circles, functions, factoring, etc.
   (c) Math terms and concepts from childhood courses or settings: fractions, decimals, geometry, etc.
   (d) Remember these closure axioms:
      i. $\mathbf{Z}$ is closed under $+$, $-$, $\times$.
      ii. $\mathbf{R}^+$ is closed under $+$, $\times$, $\div$.
      iii. $\mathbf{R}$ is closed under $+$, $-$, $\times$, non-zero $\div$.
      iv. You must use the formal definition of $\mathbf{Q}$ instead of assuming it is closed in any way.
7. Remember these proof components:
   (a) You MUST write your explicit assumptions at the outset, including universal hypotheses.
   (b) You MUST write a formal NTS claim; a "meaning" add-on is helpful but not required.
   (c) You MUST be clear that you have proved what was asked, via the "exit move" sentence.
   (d) Write in SENTENCES. Proofs are never just a string of equalities or algebraic expressions. Such things must appear in context with words telling what they represent/what you're doing with them.
8. I give PARTIAL CREDIT if you have the framework correct for your proof, even if you can't do anything past the NTS. So put on the "Thus" and "exit move" ending sentences even if you're stuck on the middle.

General Advice:

1. There will be 3-4 full proofs on the exam, with several smaller tasks.
2. Given the 50-minute time constraint, you'll find that some correct proofs require only a few sentences.
3. Some statements may be similar to examples you've already seen, but others will not be (else I'm only testing your ability to imitate, rather than actually create your own proof).
4. By their very nature, proof exams require creativity.
   (a) When required to be creative under a time constraint, it's natural to feel lost, rushed, or even blind-sided. Try not to panic, but keep working to show me as much of your skills as possible.
   (b) I like to give partial credit, so include as many *meaningful* ideas as possible, even if you're stuck.
   (c) Putting the "exit move" and other final sentences on the end typically earns some credit, regardless of whether you have a hole in the middle of your proof.
   (d) Again, practice *A LOT* (there are plenty of problems left in the text!), so that you feel comfortable choosing a proof type AND making some progress on any statement you're asked to prove.
5. If you feel you can't finish in time, **DON'T PANIC**. I often take completion time into account when determining whether an exam deserves a curve or a bonus.
6. **DO YOUR BEST** in the exam period, and let me worry about the time factor.
7. Always remember: I like to give partial credit.