Math 235 - Dr. Miller - Topics List for Final Exam, Spring 2017 - **Wednesday, May 10, 2017, 3:30-5:30 pm**

Prepare by studying these topics, in conjunction with all course materials and reading.

Non-Proof Tasks:

1. Identify hypothesis, conclusion in a conditional statement using: if-then, "trailing" if, only if (including "split"), necessary, sufficient, implies. Answers are sentences; don't keep conditional words!
2. Give SIMPLIFIED formal negations of statements that contain: conditional, if and only if (iff), and, or, for all/each/every/any, there exists/is/for some, at least, greater/less/equal, etc.
   (a) Negate quantifiers, use De Morgan's Laws where appropriate. Beware trailing quantifiers.
   (b) Remember that the negation of a conditional statement is NEVER still conditional.
3. Understand, use the notations $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, similarly for $\mathbf{Q}$ and $\mathbf{R}$, and set-builder notation.
4. Give FORMAL definitions, statements that PRECISELY control meaning AND all associated logic:
   (a) Use variables, VERBAL quantifiers, buffer words. Commas can't replace buffer words.
   (b) Give universal hypotheses or context, including set membership, associate notation, etc.
5. Give FORMAL definitions or statements of:
   (a) Rational, even, odd, Division Algorithm, divides
   (b) Union, intersection, set difference, complement, Cartesian product
   (c) Relation from A to B, relation on A, inverse of a relation, composition of relations
   (d) Domain, range, equivalence class; partition, disjoint, pairwise disjoint
   (e) Reflexive, symmetric, transitive properties; function, one-to-one, onto
   (f) Division Algorithm, Fundamental Theorem of Arithmetic (FTA)
   (g) Alternative definition of prime, Linear Combination Theorem
   (h) Transitivity of "Subsetness," Fundamental Theorem of Equivalence Relations
6. List all subsets of a given set A. Find $\mathcal{P}(A)$. Count numbers of subsets.
7. List members/shade Venn diagrams for mixed $\cup, \cap, \setminus, \times$ complement among 3 sets.
8. Given a formula/verbal description for a relation or function, list some/all ordered pairs.
9. Create sets of ordered pairs, digraphs, arrow diagrams that are/aren't or have/lack combinations of:
   (a) My choice of domain, range, number of elements (ordered pairs)
   (b) Relations, functions, one-to-one, onto, reflexive/not, symmetric/not, transitive/not
   (c) Inverses, compositions, ON a set versus from one set to another
10. Given a set of ordered pairs, digraph, arrow diagram, formula, or verbal description of a relation, identify:
    (a) Domain, range, inverse, composition
    (b) Whether it is reflexive, symmetric, transitive - explain informally
    (c) Whether it is a function, one-to-one, onto - explain informally
11. Given an equivalence relation, find $\overline{x}$ for my choice of x; find ALL sets in A/R.
12. Identify whether a collection of sets is a partition or not; create an example that is/is not; explain.

Proof Tasks: (One induction proof may be assigned as a hand-in.)

1. Write proofs using direct, cases (including "wlog"), "or conclusion," contrapositive, contradiction.
2. Write proofs by example/cex, "for all," $\subseteq$, $\not\subseteq$, $=$ sets, identities.
3. Prove whether a given relation/function is/is not reflexive, symmetric, transitive , one-to-one, onto.
4. Statements may be about even, odd, rational, remainders, divisibility, linear combinations, etc.
5. Basic concepts are fair game: graphs, lines, circles, intervals, absolute value, equations/inequalies, etc.
6. Statements may be about sets involving $\emptyset$, $\subseteq$ or $=$, $\cup$, $\cap$, $\times$, $\setminus$, complement, etc.
7. Statements may be about relation properties (#17 on last HW), partitions, functions.
8. Newer proofs in the course have required sequential use of different styles WITHIN. Be aware!
9. Remember these proof components:
   (a) You MUST write your explicit assumptions at the outset, including universal hypotheses.
   (b) You MUST be clear that you have proved what was asked, via the "exit move" sentence.
   (c) You must write in SENTENCES. **Precise** notation and math grammar will be required.
   (d) Choice of style may be mine or yours; be able to decide quickly when the choice is up to you.
10. There may be shorter tasks where I ask you only to write what we assume and what we "NTS" for MY choice of proof, and then stop. beginenumerate
11. Given a statement, set up only the first line and NTS for your choice of a correct style.
12. Prove MY choice of direction ("if" vs. "only if") in $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$