Math 235 - Dr. Miller - Topics List for Final Exam, Spring 2018 - **Wednesday, May 9, 2018, 3:30-5:30 pm**

Prepare by studying these topics, in conjunction with all course materials and reading.

Non-Proof Tasks: (Know and freely use notations $\mathbf{Z}$, $\mathbf{Z}^+$, $\mathbf{Z}^-$, similarly for $\mathbf{Q}$ and $\mathbf{R}$, and intervals.)

1. Write/rewrite a conditional statement - or its converse, inverse, contrapositive - using: if-then, "trailing" if, only if (including "split" only if), necessary, sufficient, implies, when/whenever.
2. Convert between universal, conditional, universal conditional on your own initiative: it's helpful in proofs.
3. Writing verbal negations will NOT be specifically asked, but use them freely and precisely in proofs.
   (a) Simplifying negations makes proving easier (especially cex). Beware trailing "if" or quantifiers.
   (b) Stmts may use: conditional, if and only if (iff), and, or, for all/each/every/any, there exists/is, etc.
   (c) Remember that the negation of a conditional statement is NEVER still conditional.
4. Understand a set defined using set-builder notation. List a few elements, write how to read it out loud. (This last is a new *task* but not new *knowledge* that I think will help you - see me for practice.)
5. FORMALLY state definitions, results: (MEMORIZING perfectly controls logic and helps your proofs!)
   (a) Rational, even/odd, divides, Linear Combination Thm., Division Algorithm, FTA
   (b) Alternative defn of prime, my choice of version (set/stmt) of PMI, Transitivity of "Subsetness"
   (c) Union, intersection, set difference, complement, Cartesian product, power set, subset/not a subset
   (d) Relation from A to B/on A, function, one-to-one, onto, reflexive, symmetric, transitive, anti-symmetric
   (e) Equivalence class $[x]$, Fundamental Theorem of Equivalence Relations (FTEqR), partition, disjoint
   (f) Good formal statements use variables, VERBAL quantifiers, and usually have universal hypotheses.
6. Beware algebra errors, especially factoring, variable exponents, inequality sign issues, non-zero division.
7. Given a digraph, set of ordered pairs, formula, arrow diagram, or verbal description of a relation, identify:
   (a) Whether it is reflexive, symmetric, transitive, anti-symmetric; explain informally.
   (b) Whether it is an equivalence relation, a partial order (reflexivity required); explain informally.
   (c) Whether it is a function, one-to-one, onto; explain informally. (Recall that digraph $\neq$ arrow diagram.)
8. Create sets of ordered pairs, digraphs, arrow diagrams that are/aren't or have/lack combinations of:
   (a) My choice of domain, range, ON a set versus from one set to another
   (b) Reflexive/not, symmetric/not, transitive/not, anti-symmetric/not, function/not, one-to-one/not, onto/not
   (c) Required or forbidden specific ordered pairs/xRy, number of elements (ordered pairs)
   (d) Be aware of how false hypotheses affect relation property or function definitions.
9. Given an equivalence relation, find $[x]$ or some of its members for my choice of x.
10. Identify whether a collection of sets is a partition or not; create an example that is/is not; explain.
11. As on the "Mapping terminology" handout, identify a statement as a variation of the defn of: function/not, one-to-one/not, onto/not. (Phrases NOT used in our course – image, pre-image, f^{-1} – will NOT appear.)

Proof Tasks: All techniques covered are fair game; more than one may be needed within a single problem.

1. Proof basic requirements: state all assumptions, give exit move, write sentences (algebra alone $\neq$ proof).
2. Proofs may require MY choice of style or leave it you; practice so you can choose efficiently.
3. Remember: your NTS line can help you decide how to go on in a deep proof or one with multiple styles.
4. Statements may be about topics introduced in this course, or about familiar prerequisite concepts:
   (a) Parity, rational, remainder, divides, (all using definitions, not childhood knowledge)
   (b) Set-builder sets, set operations, empty/not empty; trig functions, derivatives
   (c) Basic algebra: equation/inequality solving, line/circle/parabola equations, absolute value, etc.
5. Lemmas are allowed: closure, Linear Combination Thm, alter. defn of prime, FTA, $\subseteq$ Transitivity, etc.
   (a) If you're unsure, ask. Set identities CAN be used as lemmas (including deMorgan's Laws for sets).
6. Prove $S \subseteq T$, $S = T$, $S \not\subseteq T$, set proofs involving operations, $\emptyset$, set-builder notation. etc.
   (a) Chain-style proofs are permitted in general, but no SET IDENTITIES will be asked on the final.
   (b) Prepare for set proofs that fit other styles, however; we can prove results that are not mere identities.
   (c) Especially on set proofs, **precise** notation and math grammar will be required:
   (d) Prove my choice of direction in "$A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$." (Lemmas allowed) Prepare for rephrasings.
7. Write elementary uniqueness proofs (easier than most on HW #10, but review #1 and #5).

8. Prove whether a given relation is/is not reflexive, symmetric, transitive, anti-symmetric.
9. **PMI, set identities, and full-blown function proofs will NOT be required on this exam.**
10. Shorter tasks may ask you to verbally describe techniques WITHOUT actual proof. Such tasks typically ask only what to assume/set-up and what we need to show when proving:
    (a) "If P, then Q" directly (in general or for a specific statement)
    (b) "If P, then Q" by contrapositive (in general or for a specific statement)
    (c) Any statement by contradiction (so remember how to negate EVERYTHING)
    (d) "If P or R, then Q" directly (in general or for a specific statement)
    (e) "If P, then Q or S" (in general or for a specific statement)
    (f) "$\forall x, P(x)$" (in general or for a specific statement)
    (g) "$\exists x, P(x)$" constructively (in general or for a specific statement)
    (h) Any TFAE statement using a circular proof
    (i) The "if" direction of a biconditional statement directly, by ctp, or by contradiction
    (j) The "only if" direction of a biconditional statement directly, by ctp, or by contradiction
    (k) Disproof for conditional or "for all" statement
11. Sometimes I have to ask for a broader description of how we prove in general:
    (a) Biconditional proofs ($\Rightarrow$ and $\Leftarrow$ are allowed to use different styles); set equality
    (b) TFAE/circular proofs (often, each section is direct); chain-style proof
    (c) Uniqueness proofs; prove f is a function, one-to-one, or onto ("how-to" IS fair game for functions!)