This problem is worth an additional 10 points added to your Exam #1 score. It is individualized so that only genuine collaboration can occur.

Assign numeric values to letters in alphabetical order: \( A = 1 \), \( B = 2 \), etc. Add the last two digit number of your A-number (on your student ID), the last two digit number of your SRU student email address, and the value for the first letter of your LAST name. If your FIRST name also begins with J or L, add 10 more. The resulting number of this process is your value of \( x \). For instance, if your A-number ends in 37, your email ends in 06, your last name starts with M=13, and your first name starts with Z, you get an \( x \)-value of 56. Write your \( x \)-value here:

\[ x = \]

On the back of this page, prove the claim below carefully and with no missing steps. Proofread your response 2-3 times to see whether you may have overlooked anything in your logic. You may see me for a hint, but I won’t give HW-type help on this one.

**Prove:** \( \sqrt{x} \) is irrational for your value of \( x \).

Say your \( x \)-value was 60.

*Proof.* Assume \( \sqrt{60} \) is rational. Then \( \sqrt{60} = \frac{a}{b} \) for some integers \( a \) and \( b \) with \( b \neq 0 \). By algebra, \( 60b^5 = a^5 \). Now \( 60b^5 \) is an integer because it’s a product of them; we want it to be greater than 1, but \( b \) might be negative. However, \( \sqrt{60} \) is positive, so we may assume (without loss of generality) that both \( a \) and \( b \) are positive. Now \( 60b^5 \) is the product of positive integers, and \( 60 > 1 \), so \( 60b^5 > 1 \). Now by the FTA, we can prime factor the number \( 60b^5 = a^5 \). On the \( a^5 \) side, the prime factor 3 appears a multiple of 5 times (the 3s appearing \( a \)'s factorization will be listed 5 times apiece to create \( a^5 \)'s factorization). On the \( 60b^5 \) side, 3s appear 1 more than a multiple of 5 times (a multiple of 5 times in \( b^5 \)'s factorization, and one additional 3 from 60’s prime factorization). This contradicts uniqueness of prime factorization from the FTA. Therefore, \( \sqrt{60} \) is irrational.