Math 304 - Dr. Miller - Exam #1, Spring 2018 - Tuesday, Mar. 27, 2018

Work on the blank paper; staple this page to the front. Point values total **100**, but this exam will be **scaled** to 150 points after grading.

1. *[1 pt]* Name the four components of an axiomatic system.

2. *[8 pts - 4 ea]* Formally state your choice of any TWO Incidence Axioms.

3. Consider this interpretation:

   - Points are the integers 3,4,5,6,7.
   - Lines are sets of the form $\{1, 2, \ldots n\}$ OR $\{n, n+1, \ldots, 10\}$ where n is one of the point values above. For instance, $\{1, 2, 3, 4, 5, 6, 7\}$ is a line, as is $\{6, 7, 8, 9, 10\}$.
   - "Lie on" means "is an element of."

   (a) *[12 pts - 4 ea]* Determine whether the interpretation passes or fails each of the THREE Incidence Axioms, and briefly justify your claim.

   (b) *[2 pts]* Give an example of ONE line and ONE point from this interpretation that seem to support the Hyperbolic Parallel Postulate.

   (c) *[4 pts]* Does the interpretation in fact satisfy that postulate overall? Explain.

4. (a) *[2 pts]* In the Klein disk, draw two parallel lines that intersect in the Cartesian plane.

   (b) *[6 pts]* Name and formally state the Parallel Postulate that the sphere $\mathbf{S}^2$ satisfies.

   (c) *[4 pts]* Formally negate the Postulate you just stated. (I'll tell you one for a deduction.)

5. (a) *[4 pts]* Formally state your choice of any ONE Betweenness Axiom.

   (b) *[12 pts - 4,8]* Formally state the definition of $\overline{AB}$, then use only that definition, Betweenness Axioms, and Modern Concepts techniques to prove that if $A * C * B$, then $\overline{AC} \subseteq \overline{AB}$.

6. (a) *[4 pts]* Formally state the (shortened) Ruler Postulate.

   (b) *[4 pts]* Formally define what "same side of line ℓ" means.

   (c) *[14 pts - 4, 10]* Formally state and prove Pasch's Theorem. (I'll state it for a deduction.)

7. (a) *[2 pts]* Spell out what the acronym CPCTC stands for.

   (b) *[5 pts]* Formally state your choice of the Segment or the Angle Copying/Construction Axiom, telling which you chose.

   (c) *[16 pts - 4, 12]* Formally state and prove ASA.