Work each of questions #1-5 on a separate sheet(s) of paper, and turn them in with your name written on each. This will enable me to grade and return parts of the assignment without delaying until all are graded. For convenience, use the text’s labels C1, C2, A1, etc., as headings for each of the parts of your proof or disproof. The set of scalars is the set $\mathbb{R}$ unless otherwise stated.

1. Consider $\mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}$ with these operations:
   
   \[(a, b) + (c, d) = (a + c, b + d), \text{ so-called “coordinate-wise” addition}\]
   
   \[\alpha(a, b) = (b, a)\]

   Determine whether this structure satisfies or fails each of the FIVE conditions pertaining to scalar multiplication in the definition of a vector space. Prove your claims rigorously.

2. Consider $\mathbb{R}^2 \setminus \{(0, 0)\}$ with these operations:
   
   \[(a, b) + (c, d) = (ac/2, bd/3)\]
   \[\alpha(a, b) = (\alpha a, \alpha b)\]

   Determine whether this structure satisfies or fails each of the FIVE conditions pertaining to vector addition in the definition of a vector space. Prove your claims rigorously.

3. Consider $\mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}$ with these operations:
   
   \[(a, b) + (c, d) = (a, b)\]
   \[\alpha(a, b) = (\alpha a, 0)\]

   Determine whether this structure satisfies or fails the FIVE commutativity, associativity, or distributivity conditions. Prove your claims rigorously.

4. Consider the set $V = \{ax^2 + bx + c \mid a, b, c \geq 0 \text{ and real}\}$ with these operations:
   
   addition is the usual addition of polynomials
   \[\alpha(ax^2 + bx + c) = \mid\alpha\mid ax^2 + \mid\alpha\mid bx + \mid\alpha\mid c\]

   Determine whether this structure satisfies or fails each of the ten conditions to be a vector space. Prove your claims rigorously.

5. Consider the set $V = \mathbb{Z}$ of vectors, the set $\mathbb{N}$ of scalars, and these operations:
   
   \[a + b = ab\], that is “addition” really means ordinary integer multiplication
   \[\alpha a = a^\alpha\]

   Determine whether this structure satisfies or fails each of the ten conditions to be a vector space. Prove your claims; in some cases, though, your “proof” should simply be a 1-sentence assertion that multiplication or exponentiation is “known” to be ________.