Math 309 - Dr. Miller - Topics List for Exam #3, Spring 2017 - Friday, Apr. 28, 2017

Bases:

1. Know and use standard bases for the familiar vector spaces $\mathbf{R}^n$, P_n, and $M_{m,n}(\mathbf{R})$.
2. State the FORMAL definitions of: nullspace, row space, column space.
3. State formal definitions of $Span(S)$, of linear combination (from earlier notes).
4. Find a basis for each of the following:
   (a) the null space of a given (real) matrix A
   (b) the row space of a given (real) matrix A
   (c) the column space of a given (real) matrix A
   (d) $Span(S)$ when given S - prepare for me to require a SUBSET of S
5. Determine whether the spans of two sets S and T are equal or not.
6. Find the nullity and/or rank of a given matrix.
7. Formally state the Rank-Nullity Theorem and informally explain why it is true.
8. Informally explain what free variables tell us about linear independence of vectors/columns in a matrix, and why this makes sense.

Eigen-"things"

1. Formally state the definition of eigenvalue, of eigenvector.
2. Tell whether a given vector is an eigenvector for a given matrix (as in notes).
3. Find - by hand - all eigenvalues of a given easy matrix.
4. Explain in 2-3 lines the technique for finding eigenvalues and WHY it works.
5. Know what the Complex Conjugate Root Theorem promises about eigenvalues.
6. Know what the Fundamental Theorem of Algebra promises about eigenvalues.
7. Find a basis for the eigenspace associated to a particular eigenvalue of a matrix.
8. Use the 5 numeric facts/short-cuts we discussed for eigenvalues:
   (a) Find missing eigenvalues from a list.
   (b) Explain what one eigenvalue of a singular matrix must be, and why.
   (c) Find some eigenvalues for A^{-1}, for A^n where $n \in \mathbf{Z}^+$.
   (d) Use Complex Conjugate Root Theorem to help when needed.