Math 309 - Dr. Miller - Topics List for Exam #3 - Friday, Apr. 27, 2018

Prepare by studying the listed content with reference to your notes, text, and graded and ungraded problems.

Definition of Vector Space, Subspace:

1. Know by name and formula the ten conditions required in the definition of vector space.
2. Given an unusual vector addition $\oplus$ and/or scalar multiplication $\odot$, determine whether MY CHOICE of the ten conditions are satisfied, as in HW.
   (a) Sets involved may be $\mathbf{R}^n$, sets of matrices, $\mathbf{C}$, or $\mathbf{P}_n$.
3. Informally list the conditions necessary to be a subspace.

Special Subspaces:

1. State the FORMAL definitions of Null Space or Span.
   (a) This includes giving hypotheses/initial set-up: what are we taking the Null Space or the Span of?
   (b) Statements such as "The Null Space is when you get the zero-vector" or "The span is all of their linear combinations" earn little to no credit.
   (c) If you are unsure, memorize the book's COMPLETE definitions.
2. As in class, confirm that $Nul(A)$ or $Span\{\vec{\mathbf{v_1}}, \vec{\mathbf{v_2}}, \ldots, \vec{\mathbf{v_p}}\}$ are subspaces.
3. A little review from earlier in the course: Determine whether a specific vector belongs to a given span, to the Null Space of a given matrix; show work.
4. For Null Space, also be able to express the entire space using set-builder notation, as earlier in the course.

Basis:

1. Know the standard bases and dimensions of the familiar vector spaces $\mathbf{R}^n$, $M_{m,n}(\mathbf{R})$, $\mathbf{P}$, $\mathbf{P}_n$.
2. Determine whether a given set is a basis for a particular vector space. Justify formally or informally as permitted.
   (a) Informal: describe why no vector in the set can be created from the others
   (b) Informal: show a specific dependency relationship.
   (c) Informal: note that the dimension of V doesn't equal the number of vectors given.
   (d) Informal: describe certain vectors in V that CANNOT be created from the given set.
   (e) Formal: Create an $n \times n$ matrix and row reduce to determine spanning and independence (a la IMT).
3. Find a basis/the dimension of the Null Space of a given matrix; show work.
   (a) Reminder: Give correct dimensions and number of pivots for a matrix.

Eigen-things:

1. State the formal definition of eigenvalue or eigenvector, including the context ("given a matrix...").
2. Remember that the number 0 can be an eigenvalue, but the vector $\vec{\mathbf{0}}$ does not count as an eigenvector.
3. Confirm whether a specific vector is an eigenvector; note the associated eigenvalue.
4. Find the characteristic polynomial or characteristic equation for a given matrix. (Know the difference!)
5. Find the eigenvalues for a given matrix; give their algebraic multiplicities.
   (a) Be able to use short-cuts for triangular matrices, for non-invertible matrices.
6. Know the relationship between geometric and algebraic multiplicity, especially for the number 1.
7. Find a basis for the eigenspace associated with a specific eigenvalue.
8. Know that eigenvectors associated with distinct eigenvalues are linearly independent.
9. Find or complete an eigenvector basis for $\mathbf{R}^n$ using a given $n \times n$ matrix A.
10. Explain why setting $det(A - \lambda I) = 0$ helps us to find eigenvalues and eigenvectors.

Similarity:

1. Remember that "similar" and "row equivalent" are very different concepts.
2. Formally state the definition of similar matrices.
3. Informally state what "diagonalizable" means.
4. Justify why similar matrices have equal determinants.
5. Justify why similar matrices have the same characteristic polynomial.
6. Know how multiplicities can help us predict whether a matrix is diagonalizable.
7. Given a diagonalized matrix, raise it to a given power.
8. **Take-home Task:** Find a diagonal matrix and similarity transform for a given matrix.

Special hint: Remember all the conditions of the IMT; they'll come in handy in lots of ways!