Memorize these precise statements for use in proofs and skills:

1. Definition of vector space, subspace, row space, linear transformation, inner product
2. Definition of $\vec{0}$, of $-\vec{x}$, $\text{Span}(S)$, $e_i$, $\text{rank}(A)$, $\vec{u} \perp \vec{v}$
3. Definition of linear independence, row space, linear transformation, orthonormal set
4. Know the dimension of $\mathbb{R}^n$, $\mathbb{R}^{m \times n}$, $\mathbb{P}_n$ for all $m, n$.
5. Know the formula for inner product in $\mathbb{R}^n$, for rotation in $\mathbb{R}^2$ or $\mathbb{R}^3$.

Computational skills in $\mathbb{R}^n$, $\mathbb{R}^{m \times n}$ and $\mathbb{P}_n$ (you may bring a TI-30-type calculator):

1. Use systems of equations to decide whether a set of vectors spans $V$. (HW #6)
2. Use systems of equations to decide whether a set of vectors is linearly independent.
3. When permitted, use alternative techniques to decide independence or spanning:
   - (a) Theorem 3.3.1 about determinants and independence
   - (b) Theorem 5.5.1 about orthogonality and independence
   - (c) Theorem 3.4.1 about dimension and dependence
   - (d) Theorem 3.4.4(i) about dimension and failure to span
   - (e) Theorem 3.4.3 about dimension, independence, and spanning
4. Given a set $S$, find a subset that is a basis for $\text{Span}(S)$, as in HW #3, and explain.
5. Find a basis for the row space of a given sparse matrix, and explain.
6. Find the matrix associated to a verbally described linear transformation.
7. Given an inner product (maybe with new notation), find the norm of a given vector.
8. Given an inner product formula, determine whether given vectors are orthogonal.
9. Create an orthonormal basis from an orthogonal one.
10. Use Theorem 5.5.2 to write one vector as a linear combination of others.

Reasoning/Definitional Skills:

1. Prove/explain whether a given structure satisfies SOME vector space conditions.
2. Prove/explain whether a given subset is or is not a subspace.
3. Given sets $S$ and $T$, prove that $\text{Span}(S)$ is a subset of $\text{Span}(T)$, as in HW #4.
4. Prove/explain whether a given formula is a linear transformation.
5. Prove/explain whether a given formula satisfies the inner product conditions.

Proofs (my choice of three of these):

1. Prove that for any finite subset $S \neq \emptyset$ of a vector space $V$, $\text{Span}(S)$ is a subspace.
2. Prove that for every vector $\vec{x}$ in a vector space $V$, $0 \cdot \vec{x} = \vec{0}$.
3. Prove that in any vector space $V$, for every scalar $\alpha \in \mathbb{R}$ we have $\alpha \cdot \vec{0} = \vec{0}$.
4. Prove that if $S$ is a spanning set with $\vec{s}_0$ dependent on $S \setminus \{\vec{s}_0\}$, then $S \setminus \{\vec{s}_0\}$ is also a spanning set. (Lemma)
5. Prove Theorem 3.4.3(i): if $\dim(V) = n$, then any set of $n$ linearly independent vectors spans $V$.
6. Prove results like this one: Given $X = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $Y = \{\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_n\}$ where $\vec{x}_1 = \vec{y}_1 + \alpha \vec{y}_2$ and all other $\vec{x}_i = \vec{y}_i$, prove that $\text{Span}(X) \subseteq \text{Span}(Y)$ or conversely.
7. Prove my choice of any part of (i), (ii), or (iii) on page 171.
8. Prove the Pythagorean Law WITHOUT leaving out steps the way the text does!
9. Prove that any orthogonal set is linearly independent.