Math 309 - Dr. Miller - Topics List for Final Exam, Spring 2017 - **Wednesday, May 10, 1:00-3:00 pm**

Prepare for the exam by studying the listed tasks with reference to this semester's materials, reading, etc.

Systems of Equations: (Convert easily between system, matrix equation, vector equation, augmented matrix.)

1. Recognize, create, explain systems or augmented matrices that have/lack certain combinations of features:
   (a) Features include: consistent/inconsistent, over/underdetermined, square/diagonal/triangular, homogeneous, no/one/infinitely many solutions, free/lead variables, coefficient vs. augmented matrix.
   (b) Some creations or combinations of features may be impossible - prepare to explain why.
   (c) Systems MUST show equations originally, but you may refer to matrices/properties when explaining.
   (d) Specifically address presence or lack of conflicts when you promise consistent vs. inconsistent systems.
2. Solve given systems by hand or technology as allowed; give solution sets in correct notation when asked.

Matrices:

1. Perform and label row operations by hand when asked, including finding REF or RREF.
2. Recognize, create, explain matrices that have/lack certain combinations of features:
   (a) Features include: dimensions, REF/RREF, square, diagonal, upper/lower triangular, transpose, elementary, symmetric, singular, invertible, row-equivalent.
   (b) Some creations or combinations of features may be impossible - prepare to explain why.
   (c) Matrices REQUIRE square brackets, but determinants, individual entries, etc. are ok in explaining.
   (d) Use matrix algebra to justify claims about these features, as in Exer. Set 1.4 #10, 11, 19.
3. Know/use notation: entries, inverse, identity matrix, 0-matrix, transpose.
4. Confirm that two matrices are inverses, including in abstract settings like p.60 #25 or p.59 #19.
5. Perform arithmetic ($A+B$, $A-B$, AB, αA, A^n, A^T) on given matrices; know when it's impossible.
6. Know that matrix multiplication doesn't commute in general; give examples that do/don't.
7. Prepare to create examples about matrix products, as in p.58 #3-4.
8. Know, use algebraic rules for matrix arithmetic in Theorem 1.4.1, including problems like p.69 #11, 12.
9. Beware the rules for $(AB)^T$ and $(AB)^{-1}$. Know additional transpose rules on p. 55.
10. Use the "augment against I, then row reduce" technique to find A^{-1} or find that it doesn't exist.
11. Find a correctly ordered sequence of elementary matrices that demonstrates row equivalence.
12. Find the inverse of a given elementary matrix, including from just a verbal description of its row operation.
13. Compute the determinant of a given matrix by hand when asked. You may use formulas or visuals.
14. Know, use determinant shortcuts for: diagonal/triangular matrix, row/column of zeros, identical row/columns.
15. Know and use the relationships for $det(AB)$, $det(A^{-1})$, $det(A^T)$, $det(E)$ for elementary matrices, $det(A)$ after a row operation, that $detA \neq 0$ if and only if A^{-1} exists.
16. Answer semi-abstract computational questions about determinants, as in p.100 #7, 9.
17. Find the nullity and/or rank of a given matrix.
18. Formally state the Rank-Nullity Theorem and informally explain why it is true.

Definition of Vector Space, Subspace: (Sets may involve R^n, sets of matrices, $\mathbf{C}$, or P_n.)

1. Understand, use the names and formal statements for A1-A8, C1, or C2.
2. Given unusual vector addition, scalar multiplication, determine whether selected C1, C2, A1-A8 hold.
3. Determine whether a given set is a subspace; justify formulaically (if yes), counterexample (if not closed).

Spanning and Linear Independence:

1. These are all essentially the same task:
   (a) Given a set of vectors, determine whether a specific other vector belongs to their span. Justify.
   (b) Write a vector as a linear combination of others, if possible. If not, explain what goes wrong.
   (c) Find, use correct notation to report coordinates of a given vector with respect to a given basis.
2. Determine whether a given set is independent, whether it spans an entire vector space. Justify.
3. Justifications may allow computational work, or they may need to be verbal:
   (a) Use of determinants is welcome, but be sure you can use MATLAB to find larger ones.

(b) Discussing under- or overdetermined systems is welcome, but address the issue of conflicts too.

(c) Solving systems outright either by hand or via an RREF matrix is also welcome.

(d) "Inspection" is welcome (example: (1,0,0) isn't in the span of $(0, 2, 0)$ and $(0, 3, 0)$ because there's no way to make a first coordinate of "1" by using vectors that only have first coordinates of 0.)

(e) The size of the set compared to another known spanning or independent set - or dimension of the entire vector space may need to be used as a justification, as in recent material. (Section 3.2-3.4 Theorems, Lemmas, and Corollaries)

4. Determine whether the spans of two sets S and T are equal or not.

Basis:

1. Know the standard bases and dimensions of the familiar vector spaces R^n, $M_{m,n}(\mathbf{R})$, $\mathbf{C}$, P_n.
2. Determine whether a given set is a basis for a particular vector space. Justify.
3. Find a basis for each of the following:
   (a) the null space of a given (real) matrix A
   (b) the row space of a given (real) matrix A
   (c) the column space of a given (real) matrix A
   (d) $Span(S)$ when given S - prepare for me to require a SUBSET of S
   (e) Kernel of a given linear transformation
   (f) Image $L(S)$ of a given linear transformation and subspace S
   (g) Eigenspace associated to a specific eigenvector of a given matrix

Eigen-"things"

1. Tell whether a given vector is an eigenvector for a given matrix (as in notes).
2. Find - by hand - dome or all eigenvalues of a given easy matrix.
3. Explain in 2-3 lines the technique for finding eigenvalues and WHY it works.
4. Use A^{-1}, A^n, Complex Conjugate Root Theorem, Fundamental Theorem of Algebra info to find some or all eigenvalues for an incomplete matrix.

Linear Transformations, Dot Products:

1. Confirm (as in the vector space conditions) whether a given formula is a linear transformation.
2. Find the matrix representation of a given linear transformation.
3. In $\mathbf{R}^n$, find the norm of a given vector, distance between vectors.
4. In $\mathbf{R}^n$, determine whether given vectors are orthogonal.
5. In $\mathbf{R}^n$, given a basis, convert to a unit basis.

You may have access to Dangries' row-operator, Babcock's row-reducer, MATLAB or Mathematica, and the computer's built-in calculator.