1. Use the specified techniques to compare each collection of fractions and arrange them in decreasing order.

   (a) Draw diagrams: $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{5}$

   (b) Draw diagrams: $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{7}{8}$

   (c) Use a common denominator: $\frac{5}{6}, \frac{9}{10}, \frac{6}{8}$

   (d) Use the meanings of numerator and denominator: $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{5}$

   (e) Use the meanings of numerator and denominator: $\frac{7}{8}, \frac{70}{81}$
   (Hint: convert 7/8 to an equivalent fraction first.)

   (f) Use the meanings of numerator and denominator: $\frac{5}{6}, \frac{51}{60}, \frac{50}{61}$

   (g) Reduce to lowest terms, then use a common denominator: $\frac{4}{6}, \frac{8}{10}, \frac{6}{8}$

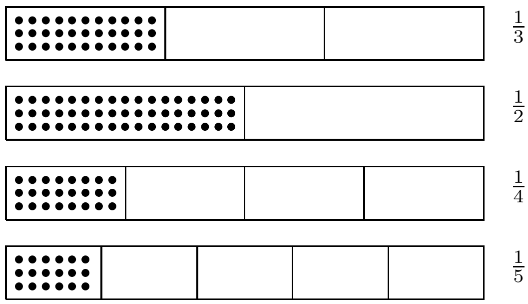
   (h) Compare to a familiar fraction first; continue as you choose: $\frac{3}{7}, \frac{5}{8}, \frac{4}{9}, \frac{7}{10}$

   (i) Demonstrate as many techniques as possible: $\frac{9}{8}, \frac{9}{7}, \frac{7}{9}$

2. Use the meanings of numerator and denominator, with equivalent fractions where permitted, in determining which of each pair below is larger. If the "meaning" method is not viable, explain why.

   (a) 3/7 vs. 4/5 (do not convert to equivalent fractions)
   (b) 3/5 vs. 4/7 (do not convert to equivalent fractions)
   (c) 5/4 vs. 7/3 (do not convert to equivalent fractions)
   (d) 50/51 vs. 49/52 (do not convert to equivalent fractions)
   (e) 50/51 vs. 51/52 (do not convert to equivalent fractions)
   (f) 50/51 vs. 101/102 (you may use equivalent fractions)

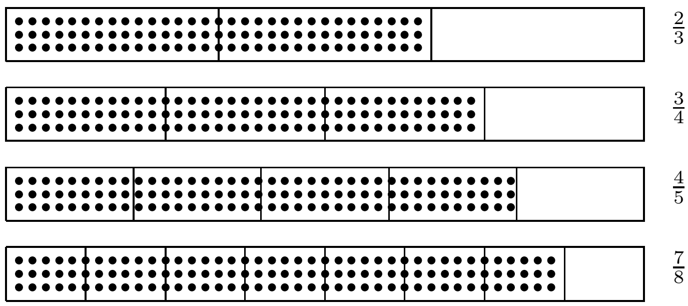
1. (a)



The pictures show that

$\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5}$

(b)



The pictures show that

$\frac{7}{8} > \frac{4}{5} > \frac{3}{4} > \frac{2}{3}$

(c) One CD is 240. Now $\frac{5}{6} \times \frac{40}{40} = \frac{200}{240}$, $\frac{9}{10} \times \frac{24}{24} = \frac{216}{240}$, and $\frac{6}{8} \times \frac{30}{30} = \frac{180}{240}$. So $\frac{9}{10} > \frac{5}{6} > \frac{6}{8}$.

(d) All keep just one piece, but halves are fatter than thirds, which are fatter than fourths, which exceed fifths. So $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5}$.

(e) Convert $\frac{7}{8}$ to $\frac{70}{80}$ to have closer numerators and denominators. They both keep the same number of pieces, but 80ths are a little fatter than 81sts, so 70/80 is the larger fraction: $\frac{7}{8} > \frac{70}{81}$.

(f) We need to convert $\frac{5}{6}$ to $\frac{50}{60}$ first. Then notice that it keeps fewer of the same-size pieces than $\frac{51}{60}$, so $\frac{5}{6} < \frac{51}{60}$. Also, it (the rewritten $\frac{5}{6}$) keeps the same number of pieces as $\frac{50}{61}$, but they're fatter pieces, so $\frac{5}{6} > \frac{50}{61}$. That means $\frac{51}{60} > \frac{5}{6} > \frac{50}{61}$.

(g) $\frac{4}{6} = \frac{2}{3}$, $\frac{8}{10} = \frac{4}{5}$, and $\frac{6}{8} = \frac{3}{4}$, so 60 is a little easier to see as a CD: $\frac{2}{3} \times \frac{20}{20} = \frac{40}{60}$, $\frac{8}{10} \times \frac{6}{6} = \frac{48}{60}$, $\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$. So $\frac{8}{10} > \frac{3}{4} > \frac{4}{6}$.

(h) $\frac{3}{7}$ and $\frac{4}{9}$ are each smaller than one half, so they're the two smallest, with cross-multiplying showing that $27 < 28$ makes $\frac{3}{7}$ the very smallest. $\frac{5}{8} \times \frac{10}{10} = \frac{50}{80}$ and $\frac{7}{10} \times \frac{8}{8} = \frac{56}{80}$ sorts out the larger two. So the order is $\frac{7}{10} > \frac{5}{8} > \frac{4}{9} > \frac{3}{7}$.

(i) Both $\frac{9}{7}$ and $\frac{9}{8}$ are more than 1, while $\frac{7}{9}$ is less than 1. Of these two larger fractions, both keep the same number of pieces, but sevenths are fatter than eighths, making $\frac{9}{7}$ largest. You can also solve by drawing diagrams, getting common denominators, or cross multiplying (BEWARE: cross-multiplying can only help you decide about two fractions at a time, never more.)

2. (a) 3/7 has fewer pieces (3 vs. 4) and they are thinner (a seventh of a pizza is skinnier than a fifth of that pizza), so 3/7 is the smaller fraction overall.

(b) We can't tell: 3/5 has fewer pieces, but they are fatter, so maybe it's bigger (its numerator thinks so) but maybe it's smaller (the denominators support that claim).

(c) 5/4 has fewer pieces, and they are a little thinner too, so 5/4 is the smaller fraction overall.

(d) 50/51 has more pieces (50 vs. 49) and they are fatter (there aren't so many 51sts in a whole as there are 52nds, making 51sts a little thicker), so 50/51 is bigger overall.

(e) This method won't work: 50/51 has fewer pieces (50 vs. 51) but they are fatter (51sts are thicker than 52nds) so there's a "stand-off": the numerators think 50/51 should be smaller (fewer pieces), but the denominators want it to be bigger (each piece worth more). Think of the coin analogy with children.

(f) Untouched, the fractions share a "stand-off": 50/51 has fewer pieces, but each piece is fatter, so we don't know whether it "wins" or not. However, 50/51=100/102. Now the fractions have same-size pieces, so the denominators create a tie and the decision is entirely up to the numerators: 100/102 has fewer pieces, so it is the smaller fraction.