Math 310 - Dr. Miller - Homework #15: Counting Problems

Leave all answers in exponent form, unless they appear very easy to simplify.

- 1. A sixth-grade teacher makes fraction cards for a game, using only the numbers 1-20 for numerators and denominators.
 - (a) How many different cards are possible if she allows numerator and denominator to be equal?
 - (b) How many cards are possible if she disallows this?
 - (c) How many cards have a 1 or 2 for the numerator? (Repeats allowed.)
 - (d) How many cards have even numbers in both positions? (Repeats allowed.)
 - (e) How many use only prime numbers? (Repeats allowed.)
- 2. A typical Illinois license plate number contains 3 letters followed by 4 digits.
 - (a) How many different license plate numbers are possible?
 - (b) How many do not repeat any letters?
 - (c) How many do not repeat any digits?
 - (d) How many repeat neither letters nor digits?
 - (e) How many have an A, B, or C as the first letter (repeated letters or digits are allowed)?
 - (f) How many use only even digits (repeated letters/digits are allowed)?
 - (g) How many do not contain the digit 2? (Repeats are allowed.)
 - (h) How many use at least one 2? (Repeats are allowed.)
 - (i) How many use your first initial at least once? (Repeats are allowed.)
- 3. A typical Wyoming license plate number has 2 letters followed by 5 digits.
 - (a) How many different license plate numbers are possible?
 - (b) How many Wyoming license plates begin with the letters WY? (Repeats are allowed.)
 - (c) How many do *not* begin with WY? (Repeats are allowed.)
 - (d) How many use the numbers 12345 in that order? (Repeats allowed.)
 - (e) How many do *not* use the digit 1 at all? (Repeated letters or digits are NOT allowed.)
 - (f) How many don't contain vowels (A,E,I,O,U)? (Repeated letters NOT allowed.)
 - (g) How many have at least one vowel? (Repeated letters NOT allowed.)
 - (h) How many don't use the letter X at all? (Repeats allowed.)
 - (i) How many use at least one X? (Repeats allowed.)
 - (j) How many use at least one odd digit? (Repeats NOT allowed.)

Math 310 - Dr. Miller - Solutions to HW #15: Counting Problems

- 1. (a) $20 \cdot 20 = 400$
 - (b) $20 \cdot 19 = 380$
 - (c) $2 \cdot 20 = 40$
 - (d) $10 \cdot 10 = 100$
 - (e) $8 \cdot 8 = 64$ (Remember, the number 1 is not a prime number.)
- 2. (a) $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 26^3 \cdot 10^4$
 - (b) $26 \cdot 25 \cdot 24 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 26 \cdot 25 \cdot 24 \cdot 10^4$
 - (c) $26 \cdot 26 \cdot 26 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 26^3 \cdot 10 \cdot 9 \cdot 8 \cdot 7$
 - (d) $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$
 - (e) $3 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 3 \cdot 26^2 \cdot 10^4$
 - (f) $26 \cdot 26 \cdot 26 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 26^3 \cdot 5^4$
 - (g) $26 \cdot 26 \cdot 26 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 26^3 \cdot 9^4$
 - (h) All minus above: $26^3 \cdot 10^4 26^3 \cdot 9^4$
 - (i) All minus those that don't: $26^3 \cdot 10^4 25^3 \cdot 10^4$
- 3. (a) $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^5$
 - (b) $1 \cdot 1 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$
 - (c) All minus above: $26^2 \cdot 10^5 10^5$
 - (d) $26 \cdot 26 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 26^2$
 - (e) $26 \cdot 25 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$
 - (f) $21 \cdot 20 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 21 \cdot 20 \cdot 10^5$
 - (g) All w/non-repeated letters minus above: $26 \cdot 25 \cdot 10^5 21 \cdot 20 \cdot 10^5$
 - (h) $25 \cdot 25 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 25^2 \cdot 10^5$
 - (i) All minus above: $26^2 \cdot 10^5 25^2 \cdot 10^5$
 - (j) All w/no repeats minus those that don't use odds: $26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 26 \cdot 25 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$