Instead of studying for spring final exams, roommates Kerry and Caty decided to go shopping for summer clothes. There were sales everywhere and the two were hoping to save a lot of money on something really spectacular for summer.

At the Old Khaki store Kerry found a really neat summer dress that was originally $80 on sale for 25% off and when Kerry went to the register, she won another 40% off on the sale price from “Register Roulette.”

“This is really great!” said Kerry, “I got an $80 dress for $36.”

“Yes, but we didn’t find anything for me,” replied Caty.

“Let’s go to Z-Mart next.”

At Z-Mart, Caty found a different but great dress also for $80. Her dress was marked 50% off and Z-Mart was having an “Everything an additional 15% off” day. “Wow, we got the same total percents off so mine should only be $36, too,” said Caty. “What a great shopping spree.”

When Caty went to the register, the clerk said her total was $34. Caty was really happy and paid her bill but Kerry was upset.

“Look, our percents off added up to the same off of the same original price, so why did I have to pay $2 more than you did. Did they figure it wrong?” complained Kerry. “I’m not leaving this mall until we figure this out. Let’s go to the food court.”

At the food court, Caty got her calculator out of her book bag and the two girls started multiplying and subtracting and multiplying some more and they just seemed to be getting all kinds of different answers that didn’t match any of the prices that they paid.

“Look,” said Caty, “when we took that math course for elementary education majors and did percents, that professor kept saying that it was faster to calculate the percent that we were paying instead calculating the percent off and subtracting. Maybe we should try that.”

“So, my 25% off means that I paid 75% of the original price and then the 40% off means I paid 60% of the sale price. So the bill for my $80 dress should be calculated by taking $80 times 0.75 times 0.60. That’s $36 which is what I paid,” said Kerry.

“And my 50% off means I paid 50% of the original price. The 15% off means I paid 85% of the sale price so my calculations should be $80 times 0.50 times 0.85 which is $34 and that is what I paid.”

Added Caty, “so I guess the stores were both right.”

“But it seems so weird that we got the same percents off and paid different amounts.”

“I bet it has to do with the fact that percent off is really a multiplication and adding the percents doesn’t work,” said Caty.

“I wonder if I could have talked them into giving me the 40% off first and then the 25% off, if it would have cost me less,” asked Kerry. “Let’s see. That would be $80 times 0.60 times 0.75 if we keep going with the multiplying by the percent we pay idea. No, it’s the same amount -- $36.”

“Of course, it has to be,” said Caty, “remember that
law about multiplication, what was it called?"

"The commutative property. If we use a variable for the price of the dress, such as $x$ then it will work for anything that we buy, then the multiplications for your dress are 0.75(0.6$x$) and 0.6(0.75$x$) and that could simplify to 0.45$x$. Wow! That means you only paid 45% of the original price."

Caty then thought, "I wonder if we changed the amounts off so they still add up to 65% off, which would give the lowest discount price."

"Huh?" asked Kerry.

"Well, I wonder if it's better to get one big and one small amount off, you know, one after the other. Or if it's better to get two moderate amounts off." Said Caty. "I think we need to look at some cases and look for a pattern. Get a placemat and we'll turn it over and make a table."

The roommates looked at two successive percents adding up to 25% and found a difference of up to 1.5%.

**TABLE ONE**

*Successive Percents Adding Up to 25%*

<table>
<thead>
<tr>
<th>1st % off</th>
<th>1st % Paying</th>
<th>2nd % off</th>
<th>2nd % Paying</th>
<th>Total % Paying</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>90% = .90</td>
<td>15%</td>
<td>85% = .85</td>
<td>.85(.90$x$) = .765$x$</td>
</tr>
<tr>
<td>20%</td>
<td>80% = .80</td>
<td>5%</td>
<td>95% = .95</td>
<td>.95(.80$x$) = .76$x$</td>
</tr>
<tr>
<td>25%</td>
<td>75% = .75</td>
<td>0%</td>
<td></td>
<td>.75$x$</td>
</tr>
</tbody>
</table>

"Let's try ones adding up to 65% like we had," suggested Caty. (Some of table 2 is not complete so that the reader can complete it.)

**TABLE TWO**

*Successive Percents Adding Up to 65%*

<table>
<thead>
<tr>
<th>1st % off</th>
<th>1st % Paying</th>
<th>2nd % off</th>
<th>2nd % Paying</th>
<th>Total % Paying</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>50% =</td>
<td>40%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>65%</td>
<td>35%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>