You must show clear work where asked to receive full credit. This exam is worth 100 points.

1. [5 pts] Correctly spell the part of a fraction that can NEVER be zero, then explain clearly why this is so, referring to the part-of-a-whole meaning of fractions.

 denominators
 It tells how many pieces to cut the whole into, but you can’t physically cut something into 0 pieces.

2. [10 pts] If the rectangle below represents 8/5 of a whole, draw a rectangle that would represent 3/4 of the same whole. You may draw intermediate diagrams if you wish. Clearly indicate your FINAL answer; you need not explain.

\[
\begin{array}{cccccc}
\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\
\frac{3}{5} = \text{one whole}
\end{array}
\]

\[
\begin{array}{c}
\frac{3}{4} \text{ of the whole is shaded.} \\
(\text{Answer is shaded})
\end{array}
\]
3. [6 pts] In the list of fractions below, CIRCLE the largest and put a BOX around the smallest. Show brief work/reasoning. You may NOT convert to decimals or percents.

\[
\frac{81}{50}, \ \frac{90}{41}, \ \frac{61}{70}
\]

improper \ \text{improper}\ \text{proper}

\[
\frac{81}{50} \times \frac{41}{41} = \frac{3321}{2050}
\]

OR

\[
\frac{81}{50} \times \frac{90}{41}
\]

so \ 3321 < 4500

or \ \frac{81}{50} < \frac{90}{41}

4. (a) [4 pts] What does the concept of "denseness" mean?

It means that between any two unequal fractions, you can always find another one.

(b) [3 pts] Illustrate this concept on the fractions 5/7 and 6/7.

\[
\frac{5}{7} \times \frac{2}{2} = \frac{10}{14} \quad \rightarrow \quad \frac{11}{14} \text{ is in between.}
\]

\[
\frac{6}{7} \times \frac{2}{2} = \frac{12}{14}
\]

5. [8 pts] Demonstrate complete "pre-cancelling" in the following computation, so that your answer is immediately in lowest terms. Show neat, clear work.

\[
\frac{50}{60} \times \frac{30}{45} \div \frac{25}{15}
\]

\[
\frac{50 \div 5}{60 \div 3} \times \frac{30 \div 15}{45 \div 15} \times \frac{25 \div 15}{3} = \frac{1}{3}
\]
6. [4 pts] One of the fractions below can be converted to a mixed number. Circle it, then demonstrate the conversion. Also circle your final answer. You may not refer to decimals or percents.

\[
\begin{array}{c}
8/320 \\
\frac{4 \text{ r. } 3}{20} \\
80 \\
\frac{3}{20}
\end{array}
\]

or

\[
\frac{83}{20}
\]

7. (a) [6 pts] Compute \( \frac{3}{8} - \frac{5}{6} \) entirely in mixed number notation. Show clear work.

\[
\begin{array}{c}
7 \frac{3}{8} \\
3 \times \frac{5}{6}
\end{array}
\]

\[
\frac{6}{48}
\]

(b) [6 pts] Multiply \( 6 \times 2\frac{1}{3} \) using the Distributive Property. Show clear work.

\[
6 \times 2\frac{1}{3} = 6 \left( 2 + \frac{1}{3} \right)
\]

\[
= 6 \cdot 2 + 6 \cdot \frac{1}{3}
\]

\[
= 12 + 2
\]

\[
= 14
\]
8. [10 pts] Draw and label a diagram representing \( \frac{5}{3} - \frac{2}{5} \). Tell how to separately see each component of the answer from your diagram.

\[ \text{19 pieces kept} \]
\[ \text{15 pieces in one whole} \]

\[ \text{Answer: } \frac{19}{15} \]

9. [4 pts - 2 each] Choose the best estimate for each computation below.

(a) \( \frac{16}{31} + \frac{8}{9} - \frac{5}{21} \)
\[ \approx \frac{1}{2} + 1 - \frac{1}{4} \]

Choices: 0 \( \bigcirc \) \( \frac{1}{2} \) \( \frac{1}{4} \) 3

(b) \( \frac{99}{31} \cdot \frac{8}{90} \)
\[ \approx 3 \cdot \frac{1}{10} \]

Choices: \( \bigcirc \) \( \frac{1}{4} \) 1 \( \frac{1}{3} \) \( \frac{1}{24} \)
10. (a) [2 pts] I’m thinking of two numbers. Their product is $\frac{1}{2}$. One of them is a little more than 10. Estimate the other.

$$\_ \times 10 \approx \frac{1}{2} \quad \frac{1}{20}$$

(b) [2 pts] Is your estimate bigger or smaller than my actual number?

(Many #s: a little > 10 times a little < $\frac{1}{20}$ to hit $\frac{1}{2}$ exactly.)

- bigger

11. [10 pts] Draw and label a diagram representing $\frac{2}{3} \div \frac{2}{3}$. Tell how to separately see EACH digit of the answer from your diagram.

$$3^{2/3}$$

each strip: $\frac{1}{3}$

How many groups of $\frac{2}{3}$ does it take to make $0 \frac{2}{3}$?

- It takes 5 full groups + part of another. That part is 1 slice, but we need 2 slices to make a complete group.
- So we have $\frac{1}{2}$ of another group.

Answer: $5 \frac{1}{2}$.
12. [8 pts] In Darin’s 4-H club, \(\frac{1}{4}\) of the members are fourth graders and \(\frac{1}{3}\) are fifth graders. The rest are sixth graders. What fraction of the club members are sixth graders? Show work. You may NOT convert to decimals or percents.

\[
\frac{1}{4} + \frac{1}{3} = ?
\]

\[
\frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}
\]

\[
\frac{1}{3} \cdot \frac{4}{4} = \frac{4}{12}
\]

Total = \(\frac{7}{12}\).

Leaves \(\frac{5}{12}\) for sixth grade.

13. [12 pts - 3 each] Write a complete number sentence that fits each clue below. If not possible, say so. You may not use decimals.

(a) The minuend is 3 and the difference is \(2\frac{1}{2}\).

\[
3 - \underline{1\frac{1}{2}} = 2\frac{1}{2}
\]

(b) The only factor is \(3/5\).

\[
\frac{3}{5} \times \underline{\frac{3}{5}} = \frac{9}{25}
\]

(c) The dividend and divisor are equal.

\[
\underline{2} \div 2 = \underline{1}
\]

(d) \(3/4\) is one addend.

\[
\underline{\frac{3}{4}} + \underline{\frac{1}{4}} = \underline{1}
\]