“Is it possible?” problem:

Is it possible for a set of five numbers in the range 0-100 to have a mean of 20 and a median of 80? Either give an example or explain why this is impossible.

No - to have a mean of 20, the scores can only total 100 points. With 80 as the median, at least three scores are 80 or higher, pushing the total to at least 240 points.

For versions asking for a mean of 20 and mode of 80, the reasoning is identical, except that we are only required to have two 80s. That still pushes the total to at least 160, which is still too high.

For versions asking for a mean of 80 and a mode of 20, we note that the total must be 400 to achieve that mean, yet with at least two 20s required, the remaining scores cannot make up the necessary 360 points to reach the needed total.

Finally, for versions asking for a mean of 80 and a median of 20, we have the same impossibility: we need a total of 400 to achieve the required mean, but now we have at least three scores of 20 or lower, and the remaining two scores are not able to make up the difference lacking from our total.

Tree problem:

For versions having 20 pine, 6 oak, and 10 maple, we get
\[
\frac{20(13.6) + 6(40.1) + 10(58.2)}{36} = \frac{272 + 240.6 + 582}{36} = \frac{1094.6}{36} = 30.4 \text{ years.}
\]

For versions having 6 pine, 20 oak, and 10 maple, we get
\[
\frac{6(13.6) + 20(40.1) + 10(58.2)}{36} = \frac{81.6 + 802 + 582}{36} = \frac{1465.6}{36} = 40.7 \text{ years.}
\]

For versions having 20 pine, 10 oak, and 6 maple, we get
\[
\frac{20(13.6) + 10(40.1) + 6(58.2)}{36} = \frac{272 + 401 + 349.2}{36} = \frac{1022.2}{36} = 28.4 \text{ years.}
\]

For versions having 10 pine, 20 oak, and 6 maple, we get
\[
\frac{10(13.6) + 20(40.1) + 6(58.2)}{36} = \frac{136 + 802 + 349.2}{36} = \frac{1287.2}{36} = 35.8 \text{ years.}
\]

Box and whisker plot:

In all versions, the 5-Number Summary was 11.9, 13.6, 18.8, 21.0, and 22.4. Label these five positions on your graph and be sure to give it a title! Some sort of scale on the number line itself is expected also.