Math 320 - Dr. Miller - HW #2, Fall 2018 - **Due Friday, Sept. 7, 2018**

1. (a) Find a proof of the equivalence of WOP and PMI (use the set-based version of PMI given in our text), and write it in your own words.

   As discussed in class on Wednesday, you will likely have a circular proof of the equivalence of WOP, PMI, and PCI (aka "Strong Induction").

   (This task is all over the web and many textbooks, so it's silly for me to require you to do it from scratch. However, it's a foundational mathematical equivalence that all math majors should learn at some point. I may require you to produce it - or one direction - on Exam #1, so be sure the proof you turn in is something you understand thoroughly.)

   (b) For efficiency, I often ask for just ONE direction of a biconditional proof on exams, but it's usually MY choice of direction. And to reinforce your basic logic skills, I often ask in some form other than the standard "if P, then Q" phrasing. Tell whether each statement below is equivalent to $WOP \Rightarrow PMI$ or to $PMI \Rightarrow WOP$.

   i. PMI isn't true if WOP isn't.
   ii. PMI can only be true if WOP is.
   iii. PMI is necessary for WOP.
   iv. A sufficient condition for PMI to be true is that WOP is true.
   v. Given the circular proof above, I can ask you about any two of the three theorems (WOP, PMI, PCI).

2. Use the Division Algorithm in proving that if n is a prime number greater than 5, then $6|(n^2 - 1)$.

3. (a) What is the largest number that produces any "crossing out" in a 1-to-1,000 Sieve of Eratostenes? Explain how you know.

   (b) What is the largest number that is NOT crossed out?

   (c) What is the LAST number that IS crossed out? Explain how you know. (Note: I do NOT mean the largest; rather, I'm asking in terms of the order in which we actually follow the Sieve's steps.)

   (d) Answer the above three questions for a 1-to-1,150 Sieve.