Always work homework on your own paper; staple the question sheet to the front with your name on it.

1. For each matrix below, write the standard $M_{n \times m}(S)$ notation for a specific set to which it belongs; however, the puzzle is that you are limited to using $\mathbb{Z}$, $\mathbb{Z}^+$, $\mathbb{Q}$, $\mathbb{Q}^+$, $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{C}$ no more than once each in this problem.

   \begin{align*}
   (a) \begin{bmatrix}
   3.3 & -1 \\
   0 & -8.76
   \end{bmatrix} & \quad (b) \begin{bmatrix}
   \sqrt[3]{2} & 1 & 1.28 \\
   1.8 & -8 & -\sqrt{8}
   \end{bmatrix} & \quad (c) \begin{bmatrix}
   3.2 & 1 & -3.3 \\
   1.8 & -8 & -3.232232223 \ldots & 0
   \end{bmatrix} \\
   \end{align*}

2. Perform the following computations in $\mathbb{C}$.

   \begin{align*}
   (a) \; & (i - 2)^3 & \quad (b) \; & \frac{8 - 3i}{2 + i} & \quad (c) \; & \frac{i - 9}{(1 - i)^2}
   \end{align*}

3. Perform the following computations if possible; if not possible, say why.

   \begin{align*}
   (a) \begin{bmatrix}
   4.2 & 8.6 & 0 \\
   5.2 & 2.1 & 8.7
   \end{bmatrix} + \begin{bmatrix}
   3.2 & 1 & 0 \\
   1.8 & 8 & 2.4
   \end{bmatrix} & \quad (b) \begin{bmatrix}
   4.2 & 8.6 & 0 \\
   5.2 & 2.1 & 8.7
   \end{bmatrix} \begin{bmatrix}
   3.2 & 1 & 0 \\
   1.8 & 8 & 2.4
   \end{bmatrix} \\
   \end{align*}

4. In this course, we will use without proof several claims listed below about closure for certain familiar sets. In this problem, however, I want you to dust off your Modern Concepts proof-writing skills and actually PROVE a small sample of these claims.

   (a) $\mathbb{Q}$ is closed under addition, subtraction, multiplication, and non-zero division. Formally prove that $\mathbb{Q}$ is closed under subtraction.

   (b) $M_{n \times m}(S)$ is closed under addition and subtraction when $S = \mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, or $\mathbb{C}$, and $M_{n \times n}(S)$ is closed under multiplication for these same sets $S$. Formally prove that $M_{2 \times 2}(\mathbb{Z}^+)$ is closed under multiplication.

   (c) $\mathbb{C}$ is closed under addition, subtraction, multiplication, and non-zero division. Formally prove that $\mathbb{C}$ is closed under non-zero division.

5. On Friday we’ll have a quick in-class quiz about which sets are closed and under which operations. The statements in Question #4 will be fair game, as will your responses to the items on the back. Practice by circling the correct response in each case.
(a) Under ordinary addition, \( \mathbb{Z} \) \( IS IS NOT \) closed.
(b) Under ordinary subtraction, \( \mathbb{Z} \) \( IS IS NOT \) closed.
(c) Under ordinary multiplication, \( \mathbb{Z} \) \( IS IS NOT \) closed.
(d) Under ordinary division, \( \mathbb{Z} \) \( IS IS NOT \) closed.

(e) Under ordinary addition, \( \mathbb{Z} \setminus \{0\} \) \( IS IS NOT \) closed.
(f) Under ordinary subtraction, \( \mathbb{Z} \setminus \{0\} \) \( IS IS NOT \) closed.
(g) Under ordinary multiplication, \( \mathbb{Z} \setminus \{0\} \) \( IS IS NOT \) closed.
(h) Under ordinary division, \( \mathbb{Z} \setminus \{0\} \) \( IS IS NOT \) closed.

(i) Under ordinary addition, \( \mathbb{Q} \) \( IS IS NOT \) closed.
(j) Under ordinary subtraction, \( \mathbb{Q} \) \( IS IS NOT \) closed.
(k) Under ordinary multiplication, \( \mathbb{Q} \) \( IS IS NOT \) closed.
(l) Under ordinary division, \( \mathbb{Q} \) \( IS IS NOT \) closed.

(m) Under ordinary addition, \( \mathbb{Q} \setminus \{0\} \) \( IS IS NOT \) closed.
(n) Under ordinary subtraction, \( \mathbb{Q} \setminus \{0\} \) \( IS IS NOT \) closed.
(o) Under ordinary multiplication, \( \mathbb{Q} \setminus \{0\} \) \( IS IS NOT \) closed.
(p) Under ordinary division, \( \mathbb{Q} \setminus \{0\} \) \( IS IS NOT \) closed.

(q) Under ordinary addition, \( \mathbb{R} \) \( IS IS NOT \) closed.
(r) Under ordinary subtraction, \( \mathbb{R} \) \( IS IS NOT \) closed.
(s) Under ordinary multiplication, \( \mathbb{R} \) \( IS IS NOT \) closed.
(t) Under ordinary division, \( \mathbb{R} \) \( IS IS NOT \) closed.

(u) Under ordinary addition, \( \mathbb{R} \setminus \{0\} \) \( IS IS NOT \) closed.
(v) Under ordinary subtraction, \( \mathbb{R} \setminus \{0\} \) \( IS IS NOT \) closed.
(w) Under ordinary multiplication, \( \mathbb{R} \setminus \{0\} \) \( IS IS NOT \) closed.
(x) Under ordinary division, \( \mathbb{R} \setminus \{0\} \) \( IS IS NOT \) closed.

(y) Under ordinary addition, \( \mathbb{R^+} \) \( IS IS NOT \) closed.
(z) Under ordinary subtraction, \( \mathbb{R^+} \) \( IS IS NOT \) closed.
(a) Under ordinary multiplication, \( \mathbb{R^+} \) \( IS IS NOT \) closed.
(b) Under ordinary division, \( \mathbb{R^+} \) \( IS IS NOT \) closed.