1. Determine whether each set below is closed under the given operation. Prove all claims.
   (a) \( T = \left\{ \begin{bmatrix} a & 5b \\ 0 & 1 \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\} \) under ordinary matrix addition
   (b) The set \( T \) above under ordinary matrix multiplication
   (c) \( V = \{ a + bi \in \mathbb{C} \mid a^2 = b^2 \} \) under complex addition
   (d) The set \( V \) above under complex division

2. Consider the operation defined on \( \mathbb{Z} \) via \( a \ast b = 2ab - 1 \).
   (a) Is \( \ast \) associative? Prove your claim.
   (b) Is \( \ast \) commutative? Prove your claim.

3. Consider the operation on \( \mathbb{Z} \) defined via \( a \ast b = 21 \).
   (a) Is \( \ast \) associative? Prove your claim.
   (b) Is \( \ast \) commutative? Prove your claim.

4. Consider the operation on \( M_{2 \times 2}(\mathbb{Z}) \) defined via
   \[
   \begin{bmatrix} a & b \\ c & d \end{bmatrix} \ast \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 0 & by \\ 1 & a \end{bmatrix}.
   
   (a) Is \( \ast \) associative? Prove your claim.
   (b) Is \( \ast \) commutative? Prove your claim.