1. (a) Give an example of a finite cyclic group of order 9, specifying its generator.
   (b) Find as many other generators as possible for your group above.
   (c) Give an example of a $U_n$ group that is cyclic, specifying its generator.
   (d) Give an example of a $U_n$ group that is not cyclic.
   (e) Explain why no $D_n$ group (recall that $n \geq 3$) can be cyclic.

2. Prove the “Cyclic Group Test”: A finite group $G$ is cyclic if and only if $G$ contains an element $a$ for which $o(a) = o(G)$.

3. Now use the Cyclic Group Test where possible to determine which of the groups below are cyclic, justifying your reasoning briefly. If you cannot even use the test, say so, and determine nevertheless whether the group is cyclic.
   (a) $S_3$
   (b) $A_4$
   (c) $D_4$
   (d) $Z_{10}$
   (e) $U_{10}$
   (f) $Z_{12}$
   (g) $U_{12}$
   (h) $Z$
   (i) $Q$
   (j) $Q \setminus 0$

4. Work these seven problems: 15.9, 15.11, 15.12, 15.17, 15.21, 15.22, 15.28.