Math 325 - Dr. Miller - Topics List for Exam #1, Spring 2016 - Wednesday, Feb. 17, 2016

Carefully study this list in conjunction with your notes and homework assignments to prepare for the exam. Studying together is also a help.

Non-proof-based Tasks:

1. Perform arithmetic on complex numbers, on small matrices, and using operations like $+_{2\pi}$ or $\cdot_{17}$.
2. Memorize the formula for the multiplicative inverse of a 2-by-2 matrix.
   (a) It's often better to distribute the front scalar THROUGH to the interior elements.
3. Perform property tasks like those on HW #1, Part 2.
4. State the FORMAL definitions of closed, associative, commutative, identity, inverse for a single element, "contains inverses for all elements."
   (a) FORMAL definitions MUST include all hypotheses and quantifiers: "e is the identity if $e * a = a = a * e$" earns very little credit. Rather, "Given an operation $*$ on a set T, $e \in T$ is the identity if $e * a = a = a * e$ for all $a \in T$."
5. Answer true-false questions based on definitions, such as Section 2 #14-16 and #24.
6. When permitted, INFORMALLY explain why a given set is/isn't closed, has/lacks an identity, or has/lacks inverses (for all its elements) under a given operation.
   (a) One or two sentences is sufficient in such settings.
7. Informally explain whether a given Cayley table has/lacks closure, commutativity, an identity, inverses for all.
8. Create or complete Cayley tables to have or lack specific qualities, such as "has an identity but isn't commutative" or "a and c have inverses, but b does not."
9. Identify familiar sets and operations as having or lacking each of the properties necessary in a group, such as "$Q \setminus \{0\}$ is not closed under addition, but it does contain a multiplicative identity," or "Subtraction is NOT associative on $\mathbf{R}$."
10. Find successful examples or counterexamples of elements passing or failing the formulas for commutativity, associativity, closure, identity, inverses, as on HW #2.
11. Content about functions, relations from Section 0 will NOT appear on this exam, nor will isomorphism.

Proof-based Tasks:

1. Use the formal definition of $\mathbf{Q}$, $\mathbf{C}$ in proofs.
2. Be attentive with set-builder notation and with set difference! Those are often the largest source of error for students on my exams.
3. **Prove** that a given set and operation have/lack my choices of characteristics of a (abelian) group: is/isn't closed, is/isn't associative, is/isn't commutative, contains an identity, contains inverses for all its elements (lacking an identity or inverses falls in the "informal" category above).
4. There's time for at most two proofs per concept (0-2 closure, 0-2 asso, etc.), and about 6 total.
5. For problems where I require a FORMAL proof, I will tell you in advance whether you are to prove that the property passes or fails.
   (a) (So only on INFORMAL problems will you have to DECIDE whether it passes or fails.)
6. As on HW, you are allowed to know about familiar operations: for instance, you are allowed to know that ordinary multiplication is associative, or that ordinary subtraction isn't commutative, that 0 is the additive identity, etc. without having to prove those claims.
7. Study by making up operations for each other and practicing proofs on them!