Math 325 - Dr. Miller - Topics List for Final Exam - Wednesday, Dec. 10, 2014, 1:00-3:00 pm

Non-proof-based Tasks:

1. Perform arithmetic, including inverting, on complex numbers or small matrices within problems.
2. Create or complete Cayley tables to have or lack specific qualities.
3. Explain whether a given Cayley table has/lacks each abelian group condition (not associativity).
4. Informally explain whether familiar sets, operations meet each condition to be abelian groups.
5. State full formal definitions of closed, associative, commutative, identity, inverses.
6. Explain informally whether a given set-up describes a subgroup, as on HW#7.
7. State formal definitions of: $M(S)$, S_n, A_n, D_n, permutation.
8. Know that composition is associative, preserves one-to-one, preserves onto.
9. Know how composition of rotations, reflections makes them interact.
10. Write members of S_n as products of disjoint cycles, as products of transpositions.
11. Remember that disjoint cycle decompositions are unique, but transposition factorizations aren't.
12. Formally state the Division Algorithm. Apply it to a given pair of integers.
13. Formally state my choice of 1-3 definitions of congruence mod n.
14. Find members of a given equivalence class mod n; give alternative names for a given class.
15. Know orders, operations, members of: $\mathbf{R}$, $\mathbf{Q}$, $\mathbf{Z}$, $\mathbf{R}^+$, $\mathbf{Q}^+$, S_n, A_n, D_n, $\mathbf{Z}_n$, U_n, $A \times B$, G/H.
16. Perform the group operation, invert, raise to powers, in the above groups.
17. Find elements of given order in these groups; find the order of an element.
18. Use listing notation to find cyclic subgroups, cosets, equivalence classes in the above groups.
19. List all left cosets of a given subgroup, all members of G/H. Give alternative names. Find index.
20. Use careful notation between ordered pairs, cosets, equivalence classes, sets, and cyclic subgroups.
21. Determine whether a given group is cyclic; explain. Be careful for infinite groups.
22. Formally state the definitions of $\langle a \rangle$, aH, $o(a)$, $[G : H]$, normal subgroup (either).
23. Formally state Lagrange's Theorem (original). State as many corollaries as I choose.
24. Know and use the fact that $o(a) = |\langle a \rangle|$ for all a in any group.
25. When $N \triangleleft G$, create the Cayley table for G/N.
26. Use left/right coset comparison to determine whether a given subgroup is normal.
27. Explain true/false statements about operations, order, cyclic subgroups, cosets, normal subgroups.

Proof-based Tasks:

1. Prove that a given familiar or new set/operation have or lack my choices of abelian group condition.
2. Prove that my choice of $\oplus$, $\odot$, coset multiplication is a well-defined operation.
3. Prove that a given subset is a subgroup, including in $A \times B$.
4. Prove part (a) or my choice of directions in part (b) of Theorem 14.3.