Math 325 - Dr. Miller - Topics List for Final Exam, Spring 2016 - Wednesday, May 4, 2016, 1:00-3:00

Carefully study this list in conjunction with your previous exams, notes, handouts, reading, and HW to prepare for the exam. Solutions to HW and exams are on the bulletin board outside my office; exam solutions are also online.

Non-proof-based Tasks:

1. Perform arithmetic on complex numbers, small matrices, and using operations like $+_{2\pi}$ or $\cdot_{17}$.
2. FORMALLY state (include hypotheses, quantifiers, etc.):
   (a) defns of closed, associative, commutative, identity, inverse for a single element, "contains inverses for all elements," closed under division, cyclic subgroup, cyclic group, permutation, even permutation, homomorphism, isomorphism
   (b) definitions of S_n, A_n, D_n, $\mathbf{Z}_n$
   (c) the three equivalent conditions to be a subgroup, the three equivalent defns of normal
   (d) set-based and the exponent-based definitions for order of an element
   (e) Lagrange's Theorem, Cauchy's Theorem, Cayley's Theorem, the Division Algorithm
   (f) short-cut conditions for recognizing that a subgroup is normal
3. Know the orders of S_n, A_n, D_n, $\mathbf{Z}_n$, and that our familiar groups related to the reals and complex numbers all have infinite order.
4. When permitted, INFORMALLY explain why a given set is/isn't closed, has/lacks an identity, or has/lacks inverses (for all its elements) under a given operation.
5. Informally explain whether a given Cayley table has/lacks closure, commutativity, an identity, inverses for all.
6. Create or complete Cayley tables to have or lack specific qualities, such as "has an identity but isn't commutative" or "a and c have inverses, but b does not."
7. Find positive powers of elements using an unfamiliar operation, as in HW.
8. Find the cyclic subgroup generated by a given element in groups we've studied.
9. List some or all generators of a given cyclic group or subgroup.
10. Recognize members of a specific S_n, A_n, D_n, the different forms for members of a given S_n.
11. Given a subgroup H in a familiar group G, find some/all of H's left cosets.
12. Give alternative names for a single coset when asked; understand, use the notation G/H.
13. Determine whether a particular collection of cosets is a group or not, using informal reasoning.
14. Compute the order of a given element in a familiar group.
15. Use Lagrange's Theorem as a short-cut in finding members of subgroups.
16. Recognize, give examples of subgroups that are normal due to a short-cut.
17. Determine informally whether a given function is a homomorphism or not, and justify.
18. Tasks above may occur in a given PRODUCT group $A \times B$ where A and B are familiar.
19. Create possible element images for a homomorphism between two given groups.

Proof-based Tasks:

1. Be attentive with set-builder notation and with set difference!
2. **Prove** that a given set and operation have/lack my choices of characteristics of a (abelian) group, that a given set (using set-builder notation) is a subgroup.
3. Prove that any subgroup of a cyclic group is cyclic.
4. Prove the various directions in our TFAE proofs about subgroups, normal subgroups.
5. Prove Lagrange's Theorem.
6. Prove that defining $(aH)(bH) = (ab)H$ gives the same result as does the set operation in G.
7. Prove general results about subgroups, normal subgroups, element order.
8. Prove that a given formula is a homomorphism.
9. Proofs about general results may take place inside direct products.