Math 325 - Dr. Miller - Topics List for Final Exam, Spring 2017 - Friday, May 12, 2017, 1:00-3:00 pm

Carefully study this list in conjunction with this semester's materials and reading.

Non-proof-based Tasks:

1. Perform various operations in $\mathbf{C}$, $\mathbf{Z}_n$, U_n, S_n, on small matrices, functions, including finding inverses.
2. Given the name of a familiar GROUP, know its order and what operation it must use to BE a group.
3. Find some or all members of $\mathbf{Z}_n$, U_n, S_n, A_n, D_n for small $n \in \mathbf{Z}^+$.
4. Describe, recognize some/all members of S_n, A_n, D_n by general visual appearance (as on HW).
5. Informally explain whether a given Cayley table has/lacks: closure, commutativity, identity, inverses.
6. Create or complete Cayley tables to have or lack specific qualities. Be prepared to explain.
7. Give FORMAL definitions or statements, which must include all hypotheses and context:
   (a) Closed, associative, commutative, identity, inverse, closed under division, U_n
   (b) Cyclic group vs. cyclic subgroup, permutation (on a set S), S_n, A_n, D_n
   (c) $o(a)$ (including infinite), $o(G)$, congruent mod H, left coset, subgroup (one or all defns)
   (d) Division Algorithm, Well-Ordering Principle, Cayley's Theorem, Lagrange's Theorem
8. Identify familiar sets, operations (including $\mathbf{Q} \setminus \{0\}$, $\mathbf{R}^+$) as having or lacking group properties.
9. Informally explain whether a given set, operation $*$ have/lack my choice of group properties.
   (a) This may require *finding* an identity or inverse, but you'd not have to prove.
10. Evaluate exponents under unusual operations; remember regular exponent rules!
11. Informally identify and justify subgroup relationships, as in HW, Exam #2. Beware failed "big groups."
12. Find the cyclic subgroup generated by a given $a \in G$ (use Lagrange), some or all generators.
13. Determine whether a given familiar GROUP is cyclic or not. Justify.
14. Write, identify $p \in S_n$ in standard notation or as product of transpositions, including non-uniquely.
15. Find elements of a given order in familiar groups; find the order of an element.
16. Use, understand cycle notation and verbal descriptions of members/products of members in D_n.
17. Give examples of elements that are/are not congruent mod a given H (in a given group G).
18. Find all elements in the equivalence class of a given element mod H.
19. List all the left cosets in a given setting. Give alternative names when asked.
20. Know and use the fact that cosets partition, that left and right cosets can be unequal yet also non-disjoint.

Proof-based Tasks:

1. **Prove** that a given set and operation $*$ have/lack my choices of group properties.
2. Beware sets defined using $\mathbf{Q}$, $\mathbf{C}$, set-builder notation, set difference!
3. Prove simple results directly from definitions, such as uniqueness of identity, properties of inverses, etc.
4. You are allowed to know properties of the sets $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$ without having to prove those claims.
5. Prove MY choice of parts that any subgroup of a cyclic group is cyclic.
6. Prove that a given set is/is not subgroup, your choice of styles.
7. Prove simple results based on definitions of group, subgroup, cyclic group.
8. Prove that $o(a) = \infty$ via the exponent definition iff $o(a) = \infty$ via the cardinality definition.
9. Prove results about the order of generic elements, as on recent HW.
10. Prove that $H \sim aH$ for all $a \in G, H < G$; prove Lagrange's Theorem.