Math 326 - Dr. Miller - Topics List for Exam #1, Fall 2017 - Monday, Oct. 2, 2017

Non-Proof Tasks:

1. Formally state:
   (a) Lagrange's Theorem, Fundamental Theorem of Homomorphisms (short in-class version)
   (b) Definition of order of a group or subgroup, of index of a subgroup
   (c) One or all (as I allow) of the definitions of normal subgroup
   (d) Definition of center of a group, kernel of a homomorphism
2. Given a subgroup H of a group G, find all the distinct left cosets of H.
   (a) Know, use (from Abs. Alg. I) that such cosets partition G, that they are all the same "size."
3. Use your choice of definitions or shortcuts to confirm whether a given subgroup is normal.
4. Know that coset multiplication uses "shortcut" definition $aHbH = abH$ ONLY when H is normal.
5. Create Cayley tables for the cosets of a given H, equivalently, for G/H. (It's the same task.)
6. Be consistent with your names for cosets; give alternative names ONLY if I ask.
7. Recognize abelian or cyclic groups in concrete examples.
8. Find several subgroups of a given group, including ones of different orders.
9. Find elements that do/do not belong to the center of a given group.
10. Prepare for $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{Q}^+$, $\mathbf{R}^+$, S_n, A_n, D_n, $\mathbf{Z}_n$, U_n.
11. Confirm whether a given formula is a group homomorphism.
12. Find and justify one or more homomorphisms - perhaps nontrivial - between given groups.
    (a) Analyze $x - y$ tables to create homomorphisms or explain why a given example must fail.
    (b) Short-cut by using concepts about element or kernel order.
13. Use Cayley tables as in HW #3, Problem #3 to recognize rings/fields/failures.
14. Give examples of rings that are/aren't commutative, are/aren't finite, are/aren't fields, have/lack unity, have/lack units, and COMBINATIONS of these features. If not possible, explain why.
15. Answer true/false questions about rings, fields, etc., as on HW #3.

Proof Tasks:

1. Prove that the center of a group, that the kernel of a homomorphism are subgroups.
2. Prove that the center of a group, that the kernel of a homomorphism are normal.
3. Prove that $\phi(e) = e'$, that $\phi(a^{-1}) = [\phi(a)]^{-1}$, that $o(\phi(a)) \mid o(a)$.
4. Prove my choice of Theorem 18.8 part 1 or part 2. (not part 3)

Material from Monday, Sept. 25, is NOT explicitly on the exam.