Math 326 - Dr. Miller - Topics List for Exam #3, Fall 2017 - Wednesday, Dec. 6, 2017

Thanks to subrings, this exam requires you to re-use information and skills introduced before Exam #2, so back up and re-study your notes from that point of the semester as well!

Non-Proof Tasks:

1. State the formal definitions of subring, ideal. Concise versions are fine.
2. In familiar rings ($\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$, $\mathbf{Z}_n$, $M_{n\times n}$), identify subrings/not, ideals/not.
3. Create, identify examples of rings, subrings having combinations of these qualities:
   (a) Are/aren't: commutative, integral domain, field, finite
   (b) Have/lack: unity, some/all/no units, some/all/no zero divisors
   (c) Especially beware how unity behaves in rings vs. subrings.
   (d) Prepare to justify informally, or to explain (informally) when impossible.
   (e) Since Exam #2 (on rings) didn't require any such example problems, I might sometimes ask you about JUST a ring - no subring also - on these tasks.
4. State the formal definition of characteristic. Find $char(R)$ for a given ring.
5. Find additive, multiplicative order (*addo* vs *multo*) of given elements in a ring.
6. Find rings, elements of given order, *char*, *addo*, *multo*. Beware/explain impossibles.
7. State formal definitions of these types of closure (review from Abstract I!):
   (a) Just plain "closed" (that is, the kind with no special operation, just $*$)
   (b) Under division, under subtraction, under inside-outside multiplication
   (c) Use good hypotheses, including having a group or ring, for the last three.
   (d) Use correct abstract algebra notation for closed under division, subtraction.
   (e) In particular, in rings "subtraction" doesn't mean $a - b$; it means $a + (-b)$.
8. Create, identify sets and subsets that are/aren't closed in the above ways.
   (a) Review the S_n, A_n, D_n, and U_n *groups* for extra examples to practice on.
9. Formally state the Division Algorithm for polynomials.
10. Demonstrate polynomial long division in $\mathbf{R}[x]$ and $\mathbf{Z}_p[x]$ with p prime (yes, we can).
11. Informally state/describe the inequalities for degree of a sum, of a product in $R[x]$.
12. Give examples of R and $f, g \in R[x]$ where the degrees of f, g, $f+g$, fg have given values or comparisons ($=$, $>$, $<$). Recognize, explain if not possible.
13. Example-type questions on the exam may be worded in different ways, such as:
    (a) If possible, give an example of.... If not possible, explain.
    (b) Is it possible...? Explain/justify.
    (c) Must ... happen? Explain/justify.
    (d) True or false...? Explain/justify.
    (e) Identify as always, sometimes, or never true...

Proof Tasks:

1. Prove short results about *addo*, *multo*. Use Lagrange's Theorem as needed.
2. Formally prove that a given subset is/isn't a subring, an ideal.
3. Given a confirmed subring, prove whether ALSO: commutative, integral domain, field, has unity.
   (a) Again, this material was first introduced before Exam #2, so refresh your memory!
4. Prove short results about characteristic, as in text reading.
5. Prove existence in the Division Algorithm, fully or my choice of starting point.
6. Prove the uniqueness portion of the Division Algorithm.