Math 326 - Dr. Miller - Topics List for Final Exam, Fall 2017 - Friday, Dec. 15, 2017, 1-3pm

*I will allow an early start at 11am - in our classroom - for those with other end-of-semester obligations, but:*
*(*) 11am and 1pm are the ONLY start times; I allow 2 hours and 15 minutes for the exam.*
*(*) No one may leave the room, use phone/text/internet, etc. until everyone has arrived.*

**Non-Proof Tasks:** *Familiar groups:* $\mathbf{Z}, \mathbf{Q}, \mathbf{R}, \mathbf{Q}^+, \mathbf{R}^+, S_n, A_n, D_n, \mathbf{Z}_n, U_n$ // *Familiar rings:* $\mathbf{Z}, \mathbf{Q}, \mathbf{R}, \mathbf{C}, \mathbf{Z}_n, M_{n\times n}$

1. Recall cyclic subgroup notation, find elements, but BEWARE: not all subgroups are cyclic ones!
2. Formally state these definitions, including necessary hypotheses:
   (a) Order of a group, subgroup, ring; index of a subgroup; characteristic of a ring
   (b) Normal subgroup (one or all defns), center of a group, kernel of a homomorphism
   (c) Additive, multiplicative order of an element (assuming 0, 1 already known identities)
   (d) EACH condition in a ring: closed, commutative, associative, identity, inverses, distributive
   (e) Commutative ring, ring with unity, integral domain, field (all assuming a ring to start with)
   (f) Unit, zero divisor, irreducible element, unique factorization domain
   (g) Subring, ideal; four types of closure, including inside-outside
3. Formally state Lagrange's Thm, Fundamental Thm of Homomorphisms, Division Algorithm in $F[x]$.
4. Given a subgroup $H$ of a group $G$, find $H$'s distinct left cosets (partitioning, size concepts can help).
5. Use your choice of definitions or shortcuts to confirm whether a given subgroup is normal.
6. Create Cayley tables for the cosets of a given $H$, equivalently, for $G/H$. (It's the same task.)
7. Confirm whether a given formula is a group or RING (*) homomorphism. **[(*) is new!]**
8. Find and justify one or more GROUP homomorphisms - perhaps nontrivial - between given groups.
9. Create, identify, justify examples or impossibility of rings, subrings having combinations of these qualities:
   (a) Are/arent: commutative, integral domain, field, finite
   (b) Have/lack: unity, some/all/no units, some/all/no zero divisors
10. Find, justify examples or impossibility in tasks on units, zero divisors, factoring elements or polynomials.
11. Identify/AVOID bad elementary algebra in equation or factoring tasks in non-fields/non-integral domains.
12. In familiar rings, identify subrings/not, ideals/not; find $char R$, $addo(a)$, $multo(a)$.
13. Find rings, elements of given order, char, addo, multo. Beware/explain impossibles.
14. Demonstrate polynomial long division in $\mathbf{R}[x]$ and $\mathbf{Z}_p[x]$; find polynomial solutions in $\mathbf{Z}_n[x]$.
15. State and support with examples the inequalities for degree of a sum, of a product in $R[x]$.
16. Example-type questions on the exam may be worded in different ways, including true/false variations.

**Proof Tasks:** *Be careful with alternative wording or proof of just ONE part of an "iff."*

1. Prove that the center of a group, that the kernel of a homomorphism are subgroups and/or normal.
2. Prove that $\phi(e) = e'$, that $\phi(a^{-1}) = [\phi(a)]^{-1}$, that $o(\phi(a)) \mid o(a)$ when $\phi$ is a GROUP homomorphism.
3. Prove, for all $a$ and $b$ in a RING, that $0a = 0$, that $-(ab) = (-a)b$, and that $n \cdot 1 = 0$ implies $n \cdot a = 0$.
4. Prove that a given set, maybe with unusual operations, satisfies my choice of conditions to be a ring.
5. Given a confirmed ring or subring, prove whether ALSO: commutative, integral domain, field, has unity.
6. Formally prove that a given subset is/isnt a subring, an ideal.
7. Prove/disprove short results (familiar or new) about zero divisors, units, integral domains, fields.
8. Prove that every finite integral domain is a field (Theorem 19.11).
9. Prove short $R[x]$ results, using precise operation definitions. Example: "$R[x]$ is commutative iff $R$ is."
10. Prove short results about addo, multo, char. Use Lagrange's Theorem as needed.

*Next Monday's new material on roots of* $\mathbf{R}[x]$ *polynomials will be assessed in an* **optional** *HW - to drop TWO.*